Systemic Risk and Market Liquidity*

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Abstract

This paper studies a model where investors' systemic risk-taking is driven by their need for market liquidity. By investing in the same asset of systemic risk, investors can expect homogeneous returns and thereby limit their private information on asset qualities. This mitigates adverse selection and fosters asset liquidity. Such liquidity creation, however, results in systemic risk: When the asset experiences a loss, all investors become stressed at the same time. Herding therefore presents a trade-off between systemic risk and liquidity creation. The model also suggests that systemic risk and leverage are mutually reinforcing: Investing in a systemic-but-liquid asset increases collateral value and debt capacity. Moreover, investors leveraged with short-term debt will find the systemic-but-liquid asset attractive for reducing the risk of runs. The paper offers an explanation of why banks collectively exposed themselves to mortgage-backed securities prior to the crisis, and why the exposure grew when banks were increasingly leveraged using wholesale short-term funding.

Keywords: Systemic Risk, Market Liquidity, Leverage

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"A 1 percent probability of failure means either that 1 percent of the banks fail every year or, alternatively, that the whole banking system fails every hundred years—quite distinct outcomes. Therefore it is crucial for regulators to find ways of discouraging herding behavior by banks."

—(*Dewatripont et al., 2010, p.116*)

1   Introduction

The recent banking crisis highlights investors' herding behaviors, especially their collective exposure to real estate bubbles, as a major source of systemic risk.[1] An in-depth understanding of investors' incentives to herd is therefore critical to the design and implementation of macro-prudential regulation. In this paper, I show that facing information frictions, investors can herd in order to create market liquidity. By investing in the same asset that yields homogeneous returns, investors limit the scope of private information on their asset qualities. This reduces potential information asymmetry that would otherwise result in market illiquidity.

Ever since the seminal papers of Akerlof (1970) and Kyle (1985), it has been well recognized that market liquidity dries up if asset payoffs are information sensitive and the trading parties are asymmetrically informed. This link between information and liquidity has inspired the rich literature of security design, which emphasizes that securities must be designed information insensitive in order to be liquid. For example, debt instruments are liquid because their payoffs are constant and insensitive to private information in all non-bankruptcy states. This paper provides a new perspective: Symmetric information and market liquidity can also be created when investors herd and collectively expose themselves to systemic risks.[2]

An asset of systemic risk yields high (low) returns for all market participants when the market is in a boom (bust). A good example would be mortgage assets,[3] whose returns crucially depend on house prices. Credit risk will be low (high) for all investors if house prices keep (stop) rising. The highly correlated returns imply that asset qualities tend to be homogeneous and publicly known among investors, leaving limited scope for private information. This mitigates adverse selection and increases asset liquidity. Such market liquidity creation, however, comes at the cost of systemic risk: Since all investors are exposed to the same risk, the financial system as a whole is less diversified. When the common risk factor—the housing price for example—takes a downturn, a systemic crisis

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[1]For example, Allen and Carletti (2011) identify six sources of systemic risk. The first is common exposure to real estate bubbles.

[2]I focus on investors' *endogenous* exposure to *exogenous* systemic shocks, and thereby dispense with modeling financial contagion that can be generated by fire sales, interbank linkages, information externalities, and so on.

[3]The term is used loosely, referring to both mortgage loans and mortgage-backed securities.

will occur. From a social welfare perspective, investors' common risk exposure presents a trade-off between private liquidity creation and systemic risk. The trade-off becomes more concrete when we compare mortgage lending with bank relationship loans. Investors are more likely to be symmetrically informed of housing price movements than of the credit worthiness of relationship borrowers because firm-idiosyncratic information only accumulates over time and tends to be privately observed by the relationship banks. Therefore, in favoring market liquidity, investors can prefer mortgage lending over relationship loans, even though firm-idiosyncratic risk can be better diversified and is less affected by the aggregate risk of housing price.

Building on the framework of Diamond and Dybvig (1983), I demonstrate that the need for market liquidity can drive systemic risk exposures. Facing uncertain returns and potential liquidity shocks,[4] risk averse investors need to smooth their consumption by both diversifying across different assets and maintaining asset liquidity. For their long-term investment to be transformed into cash without large liquidation losses, the investors need a well-functioning market that is not crippled by adverse selection. In order to preserve information symmetry and market liquidity, they may voluntarily build systemic risk into their portfolios, with the cost of under-diversification compensated by liquidity insurance. As a result, systemic risk exposure emerges as an optimal choice by investors who face liquidity need and informational constraints. From this perspective, the observed exposure to mortgages and mortgage-backed assets, while apparently creating systemic risk and having been rightly blamed for the recent crisis, can be part of the second-best allocation where investors trade-off diversification and market liquidity.

I also extend the model to study how systemic risk and leverage interact, and how accommodative credit conditions affect investors' incentives to herd. First, I show that as the collateral value of liquid assets facilitates borrowing, investors holding more systemic-but-liquid assets will use higher leverage. This channel is especially relevant as banks increasingly fund themselves by wholesale instruments such as repo. By collateralizing the systemic-but-liquid assets, a bank can keep its cost of funding low (e.g., small haircut in the case of repo) and be leveraged with wholesale short-term debt. On the other hand, the ease of borrowing also contributes to systemic risk. For instance, when accommodative monetary policy keeps borrowing cost low and makes leverage attractive, investors will reshuffle their portfolios to put more weights on systemic but liquid assets. This allows them to reap the benefit of leveraging but at the same time to reduce the associated risk of bank runs. Therefore, the model suggests that herding and leverage are mutually reinforcing, and that systemic risk-taking can be especially pronounced under accommodative credit conditions.

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[4]Diamond and Dybvig (1983) models liquidity shocks as exogenous. This contrasts the fundamentals-based bank runs emphasized in Gorton (1988), Calomiris and Gorton (1991). I discuss in section 8.2 that the main result of the paper would still hold under the alternative definition of liquidity risk.

The predictions of this stylized model are consistent with two empirical observations of the recent crisis: (1) Banks that relied more on wholesale short-term funding exposed themselves more heavily to mortgage-backed securities (MBS), and (2) in the run-up to the crisis, when accommodative monetary policy kept the cost of funding low, banks' exposure to MBS grew. Based on Call Report data of 400 biggest U.S. bank holding companies, Figure 1 illustrates these empirical regularities.

Figure 1: Mutually reinforcing herding and leveraging, and the impact of monetary policy
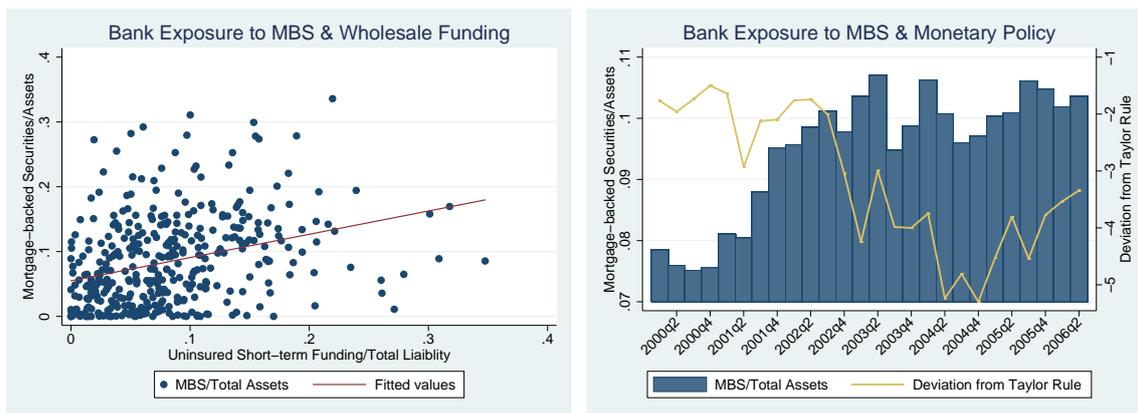
(a) Banks' MBS exposures & wholesale funding

(b) Banks' MBS exposures & monetary policy

Panel (a) plots the relationship between banks' use of wholesale funding and their exposures to mortgage-backed securities. The figure is based on a cross-section of 400 biggest U.S. bank holding companies in 2006Q2. Panel (b) depicts the U.S. monetary policy stance and banks' exposure to mortgage-backed securities over the period 2000Q1-2006Q2. The yellow line plots the deviation of effective federal fund rate from the level suggested by Taylor's rule. A negative number indicates actual federal fund rates lower than what Taylor's rule suggests. The bars show the fraction of mortgage-backed securities in banks' total assets for the same set of banks. 2006Q2 is considered as the eve of the crisis because in that quarter, the U.S. housing market took its first downturn in a decade.

This paper offers an alternative explanation for herding other than collective moral hazard. Papers such as Acharya and Yorulmazer (2007) and Farhi and Tirole (2009) emphasize that banks can coordinate to exploit public policies. The collective moral hazard results in too-many-to-fail: While individual bank failures often end up with acquisition by other banks, in a systemic crisis, regulators tend to bail out all failing banks at the cost of tax payers.[5] This explanation, however, has its limitation: If regulatory frameworks such as Basel Accord make any difference, a systemic crisis is a small probability event. It is questionable why banks make their portfolio choices in response to the payoffs in an almost-zero-probability state, particularly when public bail-outs are not always guaranteed.[6] By contrast, the current paper avoids the limitation by considering correlated risk-taking driven by information asymmetry, a friction that is not unique to crisis periods. In this regard, a related paper is

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[5]While collective moral hazard and coordination are emphasized in the recent literature, it should be recognized that information cascading has also been a prominent approach in studying of herding behaviors. Representative works include Scharfstein and Stein (1990), Banerjee (1992), Hirshleifer and Hong Teoh (2003) and Haiss (2005). For a classic survey, see Devenow and Welch (1996).

[6]Consider the bankruptcy of Lehman Brothers as an example. More importantly, the fear of not being bailed out also opens up the possibility of coordination failure in banks' collective risk-taking.

Acharya and Yorulmazer (2008). The authors focus on bank liabilities, showing that banks can herd to reduces debt holders' risk perception and thereby save on the cost of funding. Emphasizing the impact of herding on banks' asset liquidity, the current paper provides a complementary view and derives extra insights on the interaction between herding and leverage.

This paper also contributes to two strands of literature on market liquidity. First of all, it provides a simple yet novel perspective on how investors can overcome potential adverse selection and preserve market liquidity. Based on the notion of information-based liquidity,[7] the paper is most related to those on security design, represented by Gorton and Pennacchi (1990), Demarzo and Duffie (1999), DeMarzo (2005) and Dang et al. (2009). This stream of literature emphasizes that market liquidity can be created by information insensitive securities such as debts: Since debt payoffs are constant in all non-bankrupt states, private information on the returns of the underlying asset no longer leads to adverse selection. This paper suggests that, by generating homogeneous returns, assets of systemic risk also limit the scope of private information and thereby promote market liquidity. Consider again the comparison between mortgage lending and relationship loans. With default and prepayment risk of mortgage assets driven by system-wide risk factors such as house prices and interest rates, the paper offers an explanation of why mortgages and mortgage securities tend to be more liquid than other debt instruments such as corporate loans, whose firm-idiosyncratic risk leaves greater room for private information.

Second, this paper incorporates and generalizes the 'cash-in-the-market' approach of studying market liquidity. As exemplified by Allen and Gale (1994, 2005), this approach emphasizes that market illiquidity is driven by a combination of aggregate shocks and limited short-term cash supply. While retaining those features, the current paper adds to the framework private information on asset returns and adverse selection. This leads to a unified framework of studying market liquidity, one that allows both agency cost and cash-in-the-market pricing as drives of illiquidity.[8] More importantly, as investors optimally choose their exposures to systemic risk, the size of aggregate uncertainty is endogenous in the current model, which contrasts the assumption of exogenous aggregate preference shocks in the existing literature.

The paper is organized as follows. Section 2 sets up the model. I present the benchmark cases in section 3: deriving the first-best allocation and showing it can be implemented in a frictionless market. Section 4 and 5 are the core of the paper. I show that the need for market liquidity drives systemic risk exposure. In order to illustrate the main reasoning, section 4 assumes exogenous systemic risk exposures and identifies the cost and benefit of herding. Section 5 endogenizes the systemic

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[7]It should be stated that other types of agency cost, such as moral hazard or inalienable human capital, can also lead to market illiquidity and have been recognized by the literature. See Shleifer and Vishny (1992) for an example.

[8]I discuss in section 8.1 the relationship between the cash-in-the-market type of illiquidity and the agency cost based illiquidity because this conceptual issue has important policy implication such as formulating liquidity requirement.

risk exposure, showing herding happens in equilibrium. Section 6 illustrates how systemic risk and leverage mutually reinforce, with investors who invest more in the systemic but liquid assets using higher leverage, and accommodative credit conditions contributing to systemic risk-taking. Section 7 discusses empirical implications and presents corroborating empirical evidence. Section 8 discusses extensions and policy issues. Section 9 concludes.

2 Model setup

The paper analyzes the trade-off between systemic risk and market liquidity utilizing the framework of Diamond and Dybvig (1983). In particular, I modify their classic setup in four ways: (1) To model adverse selection, I introduce stochastic asset returns and private information on asset qualities. (2) To define diversification, investors are assumed to be able to invest in multiple assets with i.i.d. returns. (3) To define systemic risk, I introduce aggregate states, and assets are classified according to how sensitive their returns are to the aggregate state. I consider the more sensitive assets being systemic, because for those assets, a bad realization of the aggregate state will have a systemic consequence. And finally, (4) investors are allowed to use leverage.

2.1 Preferences

I consider a one-good, three-date ($t = 0, 1, 2$) economy inhabited by a continuum of investors $i \in [0, 1]$. Each investor has one unit of endowment to invest. They are risk averse and face liquidity shocks à-la Diamond-Dybvig: With a probability β, an investor needs to consume early and derives utility only from $t = 1$ consumption. With complementary probability $1 - \beta$, the investor is patient and consumption at date 1 and 2 are perfect substitutes. Denote period t consumption by c_t, $t = 1, 2$. A representative investor has the following two possible preferences.

$$u = \begin{cases} u(c_1) & \text{with prob } \beta \\ u(c_1 + c_2) & \text{otherwise} \end{cases}$$

There is no aggregate uncertainty on the size of the liquidity shock. In the population, a fixed fraction β of the investors will turn into early consumers.

Investors are risk averse and their utility function $u(\cdot)$ is strictly increasing and concave. In particular, I assume log-utility (unit relative risk aversion) so that the analysis focuses on the functioning of financial markets. This is because in the classic setup of Diamond and Dybvig (1983), a secondary market—despite its incompleteness concerning investors' liquidity preferences—achieves the first-best allocation under log-utility. By this assumption, I exclude the possibility of improving allocation

by contracting, and take the allocation obtained in a market with perfect information on asset qualities as a benchmark. This allows me to isolate the inefficiency due to adverse selection from the extra distortion that is caused by the market incompleteness concerning individual liquidity preferences.

2.2 Aggregate uncertainty and investment technology

There are three types of assets: a risk-free storage technology and two classes of long-term risky projects: *idiosyncratic* and *systemic*. For short, I call the risk-free storage technology *cash* and the long-term risky projects *assets*. Cash transfers one unit of date-t investment into the same amount in $t + 1, t \in \{0, 1\}$. Assets are productive, having positive NPVs, but are also risky. Their payoffs, if turn out to be low, will be less than the initial investment.

In the economy, two aggregate states, good ($s = G$) and bad ($s = B$), can occur with equal probabilities. The *idiosyncratic* and *systemic* assets differ in terms of how sensitive their returns are to the realization of the aggregate state. It is assumed that the returns of the idiosyncratic asset are not affected by the aggregate state, whereas the returns of systemic asset are perfectly correlated with the aggregate uncertainty. These rather extreme assumptions capture the definitions of "idiosyncratic" and "systemic" assets in their most simplistic forms. For concreteness, the good and bad states can be interpreted as housing market boom and bust. Then the idiosyncratic assets can be thought as banks' relationship loans, and the systemic asset as mortgage assets. While the returns of mortgage assets crucially depends on rising house prices, the performance of relationship loans is less sensitive to the housing market moves.

Formally, each unit of idiosyncratic assets generate the following $t = 2$ payoff.

$$\tilde{R}^i = \begin{cases} R_H > 1 & \text{with prob } 1/2 \\ R_L < 1 & \text{otherwise} \end{cases} \quad \text{in both states } B \text{ and } G$$

Note that the idiosyncratic returns are independent of the aggregate state: Whether the state is G or B, it is always the case that half of the population will receive high returns. By contrast, the returns of the systemic asset depend on the aggregate state and are perfectly correlated across investors. If the state happens to be good, the returns are R_H with certainty; and if the state happens to be bad, it generates returns R_L with certainty.[9]

$$\tilde{R}^s = \begin{cases} R_H > 1 & \text{in state } G \\ R_L < 1 & \text{in state } B \end{cases}$$

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[9]The model can be generalized to allow imperfect correlation: e.g., $\alpha > 1/2$ fraction of systemic assets generating high returns in the state G and $\alpha' < 1/2$ fraction of systemic assets generating high returns in state B. This allows adverse selection also for the systemic asset but would not change the results qualitatively, because the adverse selection remains lower for the systemic asset.

Take again mortgage assets as an example. This assumes that if the market turns out to be good and housing prices keep rising, all investors will gain by buying mortgages. Otherwise, all of them are going to lose.

Because of the equal probabilities of the two aggregate states, from an ex ante perspective, both systemic and idiosyncratic assets yield high returns with a probability of one half. However investing in the systemic asset involves aggregate risk exposure. Put differently, while the two classes of assets have identical unconditional distribution of returns, their conditional distribution are very different: When the state turns out bad (i.e., conditional $s = B$), all investors make simultaneous loss on their systemic asset, posing a systemic risk on the economy. It is also assumed that while all investors can choose to invest in the common systemic asset, e.g., buying mortgage-backed securities, each investor has has his private pool of idiosyncratic assets, e.g., loans to local relationship borrowers, to which the other investors cannot access.[10]

In the rest of the paper, I will refer the uncertainty of preferences "liquidity risk" and the uncertainty of asset returns "credit risk".[11] The two types of risk are orthogonal. To simplify the model, I further assume that a large number of the idiosyncratic assets, $i = 1, 2, \ldots N, \ldots$, are available for each investor. Their payoffs are identically and independently distributed. So perfect diversification can be achieved by not investing in the systemic asset and dividing long-term investment evenly across all the idiosyncratic assets. This contrasts the non-diversifiable aggregate risk in the systemic asset. In sum, there is no aggregate liquidity risk in the economy, and the existence of aggregate credit risk depends on the choice of the investors—whether they choose to invest in the systemic asset or not.

Concerning parameters, I assume the long-term assets are productive and are strictly preferred in the absence of liquidity shock, which implies

$$\frac{1}{2}\log(R_L) + \frac{1}{2}\log(R_H) > \log(1),$$

or written compactly, $\sqrt{R_L R_H} > 1$. The higher *expected utility* of risky assets implies a trade-off between liquidity and returns in choosing between cash and assets. The inequality also implies that the *expected returns* is greater than 1.

$$\overline{R} \equiv (R_H + R_L)/2 > 1$$

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[10]The assumption is motivated by the information advantage that banks often enjoy locally. Making loans in a remote and unfamiliar market can lead to winners' curses for entrants. See Sharpe (1990) and von Thadden (2004) for reference.

[11]The term 'credit risk' here is slightly abused, because the binary distributions is meant to simplify the model and does not have an unambiguous interpretation as bonds.

Given that the systemic and idiosyncratic assets have identical unconditional distribution of returns, and that the idiosyncratic credit risk is diversifiable, the systemic asset is strictly dominated from a risk-return perspective.[12] Any investment into the systemic asset would be completely driven by the need for market liquidity.

2.3   Timing, trades and information

Investors make their investment decisions at $t = 0$, choosing how much to invest in the long-term risky assets, the mixture between systemic and idiosyncratic assets, and the weights on each idiosyncratic assets. I denote the amount of risky investment by I and cash holding by $(1 - I)$. And within the risky investment, a fraction w is put on the systemic asset and $(1 - w)$ goes to the idiosyncratic assets. Last, within the portfolio of idiosyncratic assets, the weight of idiosyncratic asset i is denoted by v_i, $i = 1, 2, \ldots, N, \ldots$.

After the initial investment is made but before the intermediate date $t = 1$, information concerning states reveals. Investors learn privately their preferences and asset qualities. Upon receiving the new information, they can trade anonymously in a secondary market at $t = 1$, re-balancing their portfolios of long and short-term assets. In particular, an investor can sell his long-term assets for two reasons: He may have found himself impatient and needs to consume early, or may have found that his asset is a 'lemon' and wants to take advantage of the private information. Since the market is anonymous, a patient investor can sell only his assets of low returns without being identified.

Investors are assumed to be able to distinguish between idiosyncratic and systemic assets (e.g., telling corporate loans from mortgage loans), so two separate markets exist for the two classes of assets. Information structures differ in the two markets, with important implications for market liquidity. In the market of idiosyncratic assets, private information on asset qualities persists. Since the economy always features a half-half mixture of high and low qualities, an investor cannot to infer the others' asset qualities based on the returns he privately observed. The asymmetric information results in adverse selection and reduces secondary market prices. As a consequence, the ability of idiosyncratic assets to provide liquidity insurance is impaired. By contrast, the systemic asset promotes information symmetry, simply because the returns are homogeneous across the economy. Being aware that the whole economy is hit by the same aggregate shock of returns, an investor knows the asset quality of the others' will be identical to his. From the privately observed returns of his own asset, he is able to infer the asset quality of the others'.[13] Therefore, even if the aggregate shocks of returns entails non-diversifiable risk, its price is not distorted by asymmetric information.

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[12]In principle, the returns of idiosyncratic and systemic assets do not have to be equal. The main results of the paper qualitatively hold even if the returns of systemic asset are slightly lower.

[13]An alternative assumption is that the aggregate state is observable. For example, all investors observe the move of housing prices.

To understand the relative strength and weakness of the two types of assets, note that risk averse investors need to smooth consumption along two dimensions: (1) across different states of asset returns, and (2) across different states of liquidity preferences. While the first dimension requires diversification, the second requires a well functioning financial market that can transform long-term investment into cash without large liquidation losses. Adverse selection distorts downward asset prices and therefore is welfare reducing. This gives room for the systemic asset to improve allocations. With its returns correlated with the aggregate state, the systemic asset yields homogeneous returns for all investors and therefore limits the scope of private information. As a result, it enjoys greater market liquidity and provides more effective insurance against the liquidity risk. The exposure to the aggregate risk therefore presents a trade-off between diversification and liquidity creation.

At t = 2, long-term projects mature and their owners consume.

2.4   Leverage

The investors are allowed to lever up their investment by borrowing debt D at a risk-free rate r. The assumption of log-utility guarantees the debt to be risk-free. If the debt is risky, meaning that there are possibilities of bankruptcy, as residual claimants, the investors will derive zero payoffs in bankruptcy. For log-utility function, this implies a negative infinite expected utility. Therefore, in equilibrium the investors will never borrow so much that they are exposed to the risk of insolvency.

In determining leverage, an investor weight $t = 1$ against $t = 2$ consumption. The trade-off arises because adverse selection leads to low $t = 1$ asset prices. As the long-term risky assets are illiquid, their early liquidation can yield payoffs smaller than the initial investments. Therefore, in case of having a liquidity shock, an investor will consume less at $t = 1$ because he has to pay the early liquidation loss out of his initial endowment. But if the investor turns out to be patient, the leverage will multiply his consumption at $t = 2$. The optimal leverage therefore balances the consumption between the two states of liquidity preferences.

I interpret the initial endowment as capital, and the amount of debt D as a measurement of leverage. This simple setup captures the essence of leveraging—to increase the sensitivity of equity returns to the performance of the underlying assets. I show that with asymmetric information on asset qualities, costly liquidation—assets sold at a price lower than the initial investment—will emerge in equilibrium. By contrast, the classic Diamond-Dybvig model features an intermediate asset price equal to 1, so that investors can always recoup their initial investment. Since there is no cost for leverage, the optimal leverage is not well defined in their classic setup.

2.5 Timing

The timing of the model is depicted in the figure below.

t = 0	t = 0.5	t = 1	t = 2
1. Portfolio choices on I, w and v_i. 2. Investors lever up by borrowing D.	1. States realizes. 2. Private information on liquidity preferences arrives. 3. Private information on the asset qualities arrives.	1. Spot market opens and the investors trade. 2. The impatient investors liquidate their portfolios, repay their debts and consume. 3. The patient investors can exploit their private info by selling their 'lemons'.	1. Returns realize. 2. The patient investors repay their debts and consume.

3 Benchmarks

In this section I consider benchmark allocations. Starting with the first-best scenario where a benevolent social planner pools resources to perfectly insure both credit and liquidity risk, I show that the first-best allocation involves zero investment in the systemic asset and can be implemented in a frictionless market where there is no private information on asset qualities. These benchmarks give a clear-cut definition of herding: Any allocation that involves positive investment in the systemic asset be considered as herding and posing a systemic risk.

3.1 First-best allocation

Being able to pool resources and to assign consumption to investors, a benevolent social planner has the following optimization program.

$$\max_{w,\mathbf{v},I} E\Big[\beta \log c_1 + (1-\beta) \log c_2\Big]$$
$$s.t. \quad \beta c_1 = 1 - I \tag{1}$$
$$(1-\beta)c_2 = I[w\tilde{R}^s + (1-w)\Sigma_{i=1}^{N} v_i \tilde{R}^i]$$

The optimization can be solved in three steps. First, since the idiosyncratic credit risk is diversifiable, the social planner will avoid any unnecessary volatility in consumption caused by the risk. In the optimum, an equal weight of $v_i = 1/N$ will be placed on each of the idiosyncratic assets. In the limiting case of $N \to \infty$, the social planner perfectly diversifies the risk of idiosyncratic asset returns.

Lemma 1. *A benevolent social planner will put an equal weight $v_i = 1/N$ on each of the idiosyncratic assets. In the limiting case where $N \to \infty$, he perfectly diversifies the risk of idiosyncratic asset returns.*

Proof. See appendix A.1. □

Second, the social planner will not invest in the systemic asset. When the idiosyncratic credit risk is diversified, it is straightforward to see that the systemic asset is dominated from a risk-return perspective. It carries aggregate risk that cannot be diversified but provides no greater returns for consumption. Formally, as the social planner perfectly diversifies across all available idiosyncratic assets, he has the following optimization program that yields a zero optimal weight on the systemic asset.

$$
\max_{w,I} E\left[\beta \log c_1 + (1 - \beta) \log c_2\right]
$$
$$
s.t. \quad \beta c_1 = 1 - I \tag{2}
$$
$$
(1 - \beta)c_2 = I[w\tilde{R}^s + (1 - w)\overline{R}]
$$

Lemma 2. *A benevolent social planner will set $w^* = 0$, making no investment in the asset of systemic risk.*

Proof. See appendix A.2. □

Finally, when the benevolent social planner avoids the aggregate risk exposure and invests in a diversified portfolio of idiosyncratic assets, he avoids any uncertainty on the economy level. The credit risk is perfectly diversified; and there is no aggregate uncertainty concerning the size of liquidity shock. Formally, as $w^* = 0$, the optimization program further reduces to the following.

$$
\max_{I} \beta \log c_1 + (1 - \beta) \log c_2
$$
$$
s.t. \quad \beta c_1 = 1 - I \tag{3}
$$
$$
(1 - \beta)c_2 = I\overline{R}
$$

By pooling resources to invest, the social planer provides perfect credit and liquidity risk insurance to the investors. The first-best allocation only trades off between the productive investment and holding sufficient cash for the impatient investors to consume.

Proposition 1. *The first-best allocation entails perfect diversification of idiosyncratic credit risk and zero investment in the systemic asset. With $v_i^* = 1/N$, $w^* = 0$ and $I^* = 1 - \beta$, a benevolent social planner provides consumption $c_1^* = 1$ for the impatient investors and $c_2^* = \overline{R}$ for the patient investors.*

Proof. See appendix A.3. □

This restores the allocation of the classic Diamond-Dybvig model. Indeed, the deterministic returns in their model can be rationalized by the perfect diversification of idiosyncratic credit risk.

3.2 Allocation in a frictionless market

When investors make decentralized decisions, their investment decisions will be guided by asset prices in the secondary market. As asset prices are key to the analysis, I start with some discussion on their properties.

3.2.1 Asset prices in the secondary market

First, when the investors have symmetric information on asset returns, asset prices will reflect asset qualities. I denote by P_H^s (P_L^s) the price for an idiosyncratic asset that is going to yield returns R_H (R_L) in state s, $s \in \{B, G\}$. On the other hand, the price of systemic asset is denoted by P_s in state s. The subscript L and H are dropped because in each aggregate state there is only one possible asset quality for the systemic asset. For instance, once the aggregate state is known to be bad, there will only be systemic assets of returns R_L.

Second, the price system should exclude all arbitrage opportunities. More precisely, there should be no arbitrage between idiosyncratic assets of different qualities, nor between systemic and idiosyncratic assets. Denote R_s the returns of cash at $t = 1$ in state s, $s \in \{B, G\}$. The no-arbitrage condition requires

$$\frac{R_L}{P_B} = \frac{R_L}{P_L^B} = \frac{R_H}{P_H^B} \equiv R_B \ \text{ in state } B, \text{ and } \ \frac{R_H}{P_G} = \frac{R_L}{P_L^G} = \frac{R_H}{P_H^G} \equiv R_G \ \text{ in state } G.$$

As $\{R_L, R_H\}$ are exogenously given, the price systems in the two states are completely determined by R_B and R_G. Indeed, it is easier for us to think of returns rather than prices. While we have multiple prices for different assets in a given state, there is only one cash returns thanks to no-arbitrage principle. Once the state-specific returns of cash are known, we can pin down all the asset prices.

Third, asset prices are also state-contingent. In this model, the investment decisions are made at $t = 0$. While the supply of cash is fixed from that point of time, at $t = 1$ the aggregate risk of asset returns creates different demand for cash across states. The state-varying cash demand and the pre-fixed cash supply cause asset prices to vary across aggregate states. Therefore, any exposure to the systemic asset will result in the aggregate price volatility.[14]

Last, depending on how $t = 1$ asset prices are determined, two types of equilibrium prices can occur: (1) prices that are equal to the asset fundamental values and (2) prices that are below asset fundamental values. In the first case, the cash available at $t = 1$ exceeds the demand for cash. Asset buyers will bid until they make zero profit, so that the resulting asset prices are equal to fundamentals. In the second case, the cash available is insufficient to clear the market for prices equal to fundamen-

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[14]If the assets are instead sold to deep-pocket outsiders so that there is no longer a limited short-term cash supply, the aggregate price volatility will disappear. For example, such an setup is featured in Malherbe (2012).

tals. As a result, the buyers will bid until they use up all of their cash holdings. The equilibrium prices are such that the nominal value of assets are equal to the amount of cash available in the market, which I call generically cash-in-the-market pricing. These two cases have natural implications for equilibrium cash returns. When asset prices are equal to fundamental values, the returns of cash at $t = 1$ must be 1. On the other hand, if there is a limited amount of cash available, the long-term assets will have to be sold at a discount and the returns of cash will be greater than 1. In fact, the positive net returns provide investors the incentives to hold cash at $t = 0$. Note that in a given state, case (1) and (2) cannot co-exist, because that implies different returns of cash from buying different assets, which violates no-arbitrage conditions.

In sum, the asset prices in a frictionless market should be state-contingent, condition on asset qualities, exclude all arbitrage opportunities, and, if available cash cannot clear market for prices equal to fundamentals, reflect the limited cash supply.

3.2.2 Investment decisions

To solve for the equilibrium, note that as in the first-best case, investors will perfectly diversify across all available idiosyncratic assets. The proof uses same argument as for lemma 1. The only difference is that now imperfectly diversified idiosyncratic credit risk does not only cause volatile $t = 2$ consumption but also makes $t = 1$ consumption volatile because of the price differential between P_L^s and P_H^s.

Lemma 3. *In the market allocation, investors will put an equal weight $v_i = 1/N$ on each of the idiosyncratic assets. In the limiting case where $N \to \infty$, they perfectly diversify the risk of idiosyncratic asset returns.*

By diversifying the idiosyncratic credit risk, a representative investor solves the following maximization program, with the last two constraints being the non-arbitrage conditions.

$$\max_{w,I} E\left[\beta\left[\frac{1}{2}\log c_{1,G} + \frac{1}{2}\log c_{1,B}\right] + (1-\beta)\left[\frac{1}{2}\log c_{2,G} + \frac{1}{2}\log c_{2,B}\right]\right]$$

$$s.t. \quad c_{1,G} = (1-I) + I[wP_G + (1-w)(P_L^G + P_H^G)/2]$$

$$c_{1,B} = (1-I) + I[wP_B + (1-w)(P_L^B + P_H^B)/2]$$

$$c_{2,G} = (1-I)R_G + I[wR_H + (1-w)\overline{R}] \tag{4}$$

$$c_{2,B} = (1-I)R_B + I[wR_L + (1-w)\overline{R}]$$

$$R_L/P_B = R_L/P_L^B = R_H/P_H^B = R_B$$

$$R_H/P_G = R_L/P_L^G = R_H/P_H^G = R_G$$

Or write in the unconstrained form.

$$\max_{w,I} \left\{ \begin{array}{l} \beta\left[\frac{1}{2}\log\left[(1-I)+wI\frac{R_H}{R_G}+(1-w)I\frac{R_L+R_H}{2R_G}\right]+\frac{1}{2}\log\left[(1-I)+wIP_L+(1-w)I\frac{R_L+R_H}{2R_B}\right]\right]+ \\ (1-\beta)\left[\frac{1}{2}\log\left[(1-I)R_G+wIR_H+(1-w)I\overline{R}\right]+\frac{1}{2}\log\left[(1-I)R_B+wIR_L+(1-w)I\overline{R}\right]\right] \end{array} \right\}$$

The solution of the optimization program entails a discussion of two scenarios: (1) Asset prices are equal to the fundamental values so that net returns of cash are zero; and (2) asset prices are strictly below the fundamentals so that the limited cash supply determines prices and the net returns of cash are positive.

3.2.3   Asset prices equal to fundamentals

I first examine potential equilibrium where asset prices are equal to fundamentals. In this case, $t = 1$ cash supply is high enough, and the perfect competition among investors makes them bid until asset prices equal to fundamentals. The analysis shows that this type of equilibrium cannot exist. More precisely, asset prices cannot be equal to fundamentals in any one of the two states, nor both.

First, suppose that asset prices are equal to the fundamental values in both good and bad states. It is implied that investors can transform their risky assets into cash without incurring any cost. As a result, cash is dominated, and the investors have no incentives to hold any cash at $t = 0$. This contradicts the presumption that $t = 1$ cash supply is sufficiently high to clear the market for prices equal to fundamentals.

Lemma 4. *In equilibrium, asset prices cannot be equal to fundamental values in both good and bad states.*

Proof. See Appendix A.4. □

While lemma 4 excludes the possibility that asset prices are equal to fundamentals in both states, it is still possible that asset prices are equal to fundamentals in one state, but are ceiled by a fixed cash supply in the other state. Intuitively, prices equal to fundamentals can only occur in bad state. Because asset fundamental values are higher in good state, more initial cash holding is in need to clear the market for the prices equal to fundamentals. Formally, in both states G and B, the maximum cash supply is fixed at $(1 - \beta)(1 - I)$ at $t = 1$, because cash holding decisions are made at $t = 0$. Clearing market for prices equal to fundamentals requires cash $\beta I(wR_L + (1 - w)\overline{R})$ in state B, and $\beta I(wR_H + (1 - w)\overline{R})$ in state G. Since the latter is bigger, we have the following result.

Lemma 5. *If asset prices are equal to fundamental values in one state but are determined by cash supply in the other, it can only be the case that prices are equal to fundamentals in the bad state.*

In the surviving case of lemma 5, holding cash brings no benefit in state B, but generates positive returns in state G. From an ex ante perspective, as the realization of state is uncertain, a positive cash holding may still be rationalized. The lemma below, however, shows that this cannot arise as an equilibrium either.

Lemma 6. *Asset prices equal to fundamental values in the bad state cannot be an equilibrium. The only candidate equilibrium is the one where asset prices are determined by $t = 1$ cash supply in both good and bad states.*

Proof. See Appendix A.5. □

3.2.4 Cash-in-the-market pricing

The three lemmas above show that only cash-in-the-market type of prices can occur in equilibrium. So in both state G and B the equilibrium prices should make the nominal value of assets on sale equal to the supply of cash. Denote the liquidation value of a unit portfolio by $K_B \equiv (1-w)\overline{R}/R_B + wR_L/R_B$ in state B, and $K_G \equiv (1-w)\overline{R}/R_G + wR_H/R_G$ in state G. The market clearing conditions can be written as follows.

$$(1 - \beta)(1 - I) = \beta K_B I \quad \text{in state } B \tag{5}$$

$$(1 - \beta)(1 - I) = \beta K_G I \quad \text{in state } G \tag{6}$$

The two market clearing conditions, combined the first order conditions with respect to w and I, constitute a system of four equations and four unknowns.

To solve model, note that as the idiosyncratic credit risk is perfectly diversified, the idiosyncratic assets again dominates the systemic one, implying zero aggregate risk exposure, $w^* = 0$. The argument is similar as before: While the systemic asset does not provide extra returns, it introduces volatility for consumption. The only nuance is that with decentralized investment decisions and price mechanism, the exposure to the aggregate risk does not only cause $t = 2$ consumption volatility but also leads to $t = 1$ consumption volatility because of price differentials. Therefore, any positive exposure to the systemic asset will not part of the optimal portfolio.

Lemma 7. *Under symmetric information on asset qualities, the investors will not invest in the asset of aggregate risk, setting $w^* = 0$.*

Proof. See appendix A.6. □

In the absence of systemic risk exposure, the prices for idiosyncratic assets do not vary across states, $P_L^B = P_L^G \equiv P_L$ and $P_H^B = P_H^G \equiv P_H$, so the realization of the aggregate risk no long affects

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consumption, $c_{1,G} = c_{1,B} = c_1$ and $c_{2,G} = c_{2,B} = c_2$. A representative investor's optimization program takes the following form, with the last constraint being the no-arbitrage condition.

$$\max_I \left\{ \beta \log c_1 + (1 - \beta) \log c_2 \right\}$$

$$s.t. \quad c_1 = (1 - I) + I(P_L + P_H)/2$$

$$c_2 = (1 - I)(R_H/P_H + R_L/P_L)/2 + I\overline{R}$$

$$R_H/P_H = R_L/P_L \equiv R$$

The analysis has reduced to the classic Diamond-Dybvig model where there is no aggregate uncertainty on the economy level. For log-utility, the secondary market provides perfect liquidity insurance and achieves first-best allocation.

Proposition 2. *A market with symmetric information on asset qualities implements the first-best allocation. The investors will perfectly diversify the idiosyncratic credit risk and make zero into the systemic asset. Under the equilibrium price $P_H = R_H/\overline{R} > 1$ and $P_L = R_L/\overline{R} < 1$, they make the optimal portfolio choice where $v_i^* = 1/N$, $w^* = 0$ and $I^* = 1 - \beta$. As a result, the impatient investors consume $c_1^* = 1$ and the patient investors consume $c_2^* = \overline{R}$.*

Proof. See appendix A.7. □

The two benchmark cases give a clear-cut definition of systemic risk: Any allocation with $w > 0$ will be considered as herding and posing a systemic risk. This definition is somehow extreme, and is based on the assumption that the idiosyncratic credit risk can be perfectly diversified. Alternative optimal exposures can be derived analogously under relaxed assumptions. This extreme case, however, will best highlight the pros and cons of being exposed to an aggregate risk—its benefit of creating liquidity and its cost of reducing diversification.

4 Simplified models for illustration

To lend some intuition to the full model, I analyze in this section two polarized cases with *exogenous* aggregate risk exposure w: (1) an economy where investors are exposed to aggregate risk ($w > 0$) but have symmetric information on asset qualities, and (2) an economy where there is asymmetric information about asset qualities but the aggregate risk exposure does not exist ($w = 0$). The first case isolates the inefficiency due to imperfect diversification, identifying the cost of investing into the systemic asset. The second case isolates the impact of adverse selection on liquidity, identifying the potential benefit of investing into the systemic asset. While each case examines only

one side of the trade-off between liquidity creation and systemic risk, readers will find some most important results of the full-fledged model embedded in these simplified illustrations.

4.1   Systemic but liquid

I first show that the aggregate risk exposure does not distort asset prices and the secondary market can still provides sufficient liquidity insurance. The case deviates from the frictionless market scenario in only one aspect: The aggregate exposure, w, is now assumed to be exogenous and positive.[15] Assuming symmetric information on asset qualities, the analysis focuses on the impact of the aggregate risk exposure.

Note that all properties of asset prices in section 3.2.1 still hold: Asset prices are state-contingent, reflect asset qualities, and exclude all arbitrage opportunities. As the investors perfectly diversify the idiosyncratic credit risk, they solve the same $t = 0$ maximization as program (4), except that w is now assumed to be exogenous and positive.

To see that the aggregate risk exposure does not distort liquidation value and restores the first-best $t = 1$ consumption, note that the first order condition with respect to I yields

$$\frac{w\frac{R_H}{R_G} + (1 - w)\frac{\overline{R}}{R_G} - 1}{(1 - I) + w\frac{R_H}{R_G} + I(1 - w)\frac{\overline{R}}{R_G}} + \frac{w\frac{R_L}{R_B} + (1 - w)\frac{\overline{R}}{R_B} - 1}{(1 - I) + Iw\frac{R_L}{R_B} + I(1 - w)\frac{\overline{R}}{R_B}} = 0.$$

Recall the $t = 1$ liquidation value of a unit portfolio $K_B \equiv (1 - w)\overline{R}/R_B + wR_L/R_B$ in state B, and $K_G \equiv (1 - w)\overline{R}/R_G + wR_H/R_G$ in state G. The first order condition can be written compactly as follows.

$$\frac{K_G - 1}{(1 - I) + IK_G} + \frac{K_B - 1}{(1 - I) + IK_B} = 0 \tag{7}$$

From the discussion of section 3.2.2, we know that asset prices cannot be equal to fundamental values. In both states B and G, the asset market clear only when the nominal value of assets is equal to $t = 0$ cash holding. So the market clearing conditions (5) and (6) still hold and $K_B = K_G \equiv K$. Combining this with the first order condition (7), we know

$$K_B = K_G = 1.$$

It implies that the impatient investors consume $c_1 = (1 - I) + IK_B = (1 - I) + IK_G = 1$ in both states. The $t = 1$ consumption in the benchmark cases is restored. Furthermore, the market clearing condition $(1 - \beta)(1 - I) = \beta KI$ implies $I^* = 1 - \beta$, so the risky investment will be the same as in the

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[15]This is endogenized in the section 5. When there is asymmetric information on idiosyncratic asset returns, the optimal aggregate risk exposure will be greater than zero for its liquidity creation.

first-best. Therefore, the exposure to the aggregate risk does not distort the liquidation value of the assets. And the spot market still provides sufficient liquidity insurance.

The aggregate risk, however, results in uncertain consumption at $t = 2$ and volatile asset prices at $t = 1$. Instead of the deterministic $\bar{R}$, the investors will receive $wR_H + (1 - w)\bar{R}$ in state G and $wR_L + (1 - w)\bar{R}$ in state B. This uncertainty of $t = 2$ consumption reflects the non-diversifiable risk of the systemic asset—the cost of being exposed to the aggregate risk. Concerning the asset prices, note that the definition of K_G and K_B suggest the following returns of cash.

$$R_G = wR_H + (1 - w)\bar{R}$$

$$R_B = wR_L + (1 - w)\bar{R}$$

Because state B and G occur with equal probabilities, the expected returns of cash is equal to $\bar{R}$, exactly the same as in the frictionless market. The price of the systemic asset will be

$$P_G = \frac{R_H}{R_G} = \frac{R_H}{wR_H + (1 - w)\bar{R}} > 1 \text{ in state } G$$

$$P_B = \frac{R_L}{R_B} = \frac{R_L}{wR_L + (1 - w)\bar{R}} < 1 \text{ in state } B$$

More precisely we have

$$P_L^G < P_L^B = P_B < 1 < P_G = P_H^G < P_H^B.$$

The results are summarized in the following proposition.

Proposition 3. *When investors exposure themselves to the aggregate risk ($w > 0$) but have symmetric information on each other's asset qualities, asset liquidation value is not distorted and is kept at 1. Market exchange still implements the first-best investment ($I^* = 1 - \beta$) and the first-best $t = 1$ consumption ($c_1^* = 1$). But $t = 2$ consumption is volatile, with $c_2^* = wR_H + (1 - w)\bar{R}$ in state G and $c_2 = wR_L + (1 - w)\bar{R}$ in state B.*

The feature that limited cash supply and aggregate uncertainty lead to asset price volatility in its essence mimics the cash-in-the-market literature that emphasizes aggregate uncertainty about liquidity shocks' size. By contrast, the current model assumes aggregate uncertainty of asset returns. Since the exposure to the systemic asset is part of the investment decision, the magnitude of aggregate uncertainty will be endogenous in the model.

4.2 Adverse selection without aggregate risk exposures

Now I turn to the case where there is no aggregate risk exposure ($w = 0$) but the investors have private information on their asset qualities. The private information leads to adverse selection

problem, which will distort downward $t = 1$ asset prices and impair the liquidity insurance provided by market exchange.

The analysis takes three steps. First, I show that the equilibrium asset price must be in the range where adverse selection happens. It will be higher than the payoff of low quality assets so that patient investors will take advantage of their private information by selling their 'lemons'. Second, the assets will be sold below their fundamental values, and the asset prices will be determined by the limited cash supply. Finally, I derive the unique equilibrium price and show it is indeed distorted downwards as compared to the benchmark case.

4.2.1 Adverse selection

Once there is asymmetric information on asset returns, the asset prices can no longer condition on asset qualities. Instead, there will be a unique price $\hat{P}$ for the average quality in the market.[16] And the average quality $\hat{R}$ in turn depends on the price.

$$
\hat{R} = \begin{cases} \overline{R} & \text{if} \quad \hat{P} \leq R_L \\ (\beta R_H + R_L)/(1 + \beta) & \text{if} \quad R_L < \hat{P} \leq R_H \\ \overline{R} & \text{if} \quad \hat{P} > R_H \end{cases}
$$

When $R_L < \hat{P} \leq R_H$, the patient investors will find it profitable to sell their 'lemon' assets and adverse selection will happen. I show that the candidate equilibrium price can only be in this interval. To see this, note that any price $\hat{P} > R_H$ cannot be an equilibrium, because in that case buyers pay more than the highest possible returns and will make a loss. But it is less straightforward to exclude the case $\hat{P} \leq R_L$. Because in the framework of Diamond-Dybvig, holding cash must be rewarded by a liquidity premium and assets will have to be sold at a discount at $t = 1$, in principle, the discount can be so large that $\hat{P} \leq R_L$. In that case, the sellers of 'lemon' assets withdraw from the market and adverse selection does not happen even in the presence of asymmetric information. Lemma 8 excludes this scenario, showing that a price below R_L will be too low to induce any risky investment on the initial date, This would contradict the existence of $t = 1$ asset market in the first place.

Lemma 8. *Under asymmetric information about asset qualities, the equilibrium price can only be in the interval of (R_L, R_H). This leads to adverse selection: the patient investors sell their low quality assets in the secondary market, making the average asset quality $(\beta R_H + R_L)/(1 + \beta)$.*

Proof. See Appendix A.8. □

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[16]Because the aggregate risk exposure is assumed to be zero, the asset prices are no longer state-contingent. Since there is only one mixed quality in the market, we do not have to consider no-arbitrage conditions either.

4.2.2 Cash-in-the-market pricing

In the case of adverse selection, prices equal to fundamentals implies that $\hat{P} = \hat{R}$ and the net returns of cash is zero at $t = 1$. I show that a price $\hat{P} = \hat{R}$ will be so high that that investors choose $I^* = 1$. First of all, note that if $\hat{R} \geq 1$, assets will dominate cash: They promise better returns at $t = 2$ and can be sold for equal or more intermediate date consumption. When $\hat{P} = \hat{R} < 1$, the situation becomes more subtle. Assets still provide better $t = 2$ consumption, but if an investor turns impatient, he will have to liquidate his assets for a price lower than the initial investment. In this case, assets are ex post inferior to cash when an investor is hit by a liquidity shock. But this disadvantage diminishes exactly when the liquidity shock becomes more likely. This is because $\hat{R}$ increases in β: The average asset quality increases when more people sell for liquidity reasons. Therefore, as early liquidation becomes more likely, the cost of liquidation goes down. Overall, price $\hat{P} = \hat{R}$ is still high enough that the investors will choose not to hold any cash at $t = 1$.

Lemma 9. *Asset prices cannot be equal to the fundamentals, and $\hat{P} = \hat{R}$ cannot be an equilibrium.*

Proof. See Appendix A.9. □

4.2.3 Equilibrium price

Once $\hat{P} = \hat{R}$ is excluded as a candidate equilibrium, $R_L < \hat{P} < \hat{R}$ is the only remaining case. Thus cash yields positive net returns at $t = 1$ and the equilibrium asset prices equalize the demand and supply of cash.

Note that the patient investors will participate both in the supply and demand side of the market. For $R_L < P < \hat{R}$, a patient investor will sell his 'lemon' assets in the market. With the proceeds $\hat{P}I/2$, he commands cash $(1 - I) + \hat{P}I/2$ and buys the average quality in the secondary market. This leads to the following $t = 2$ payoff.

$$\left[(1 - I) + \hat{P}\frac{I}{2} \right] \frac{\hat{R}}{\hat{P}} + \frac{R_H}{2} I = \left(1 - I \right) \frac{\hat{R}}{P} + \frac{\hat{R} + R_H}{2} I$$

Correspondingly, the optimization program becomes

$$\max_{I} \left\{ \beta \log \left[(1 - I) + I\hat{P} \right] + (1 - \beta) \log \left[(1 - I)R + \frac{R_H + \hat{R}}{2} I \right] \right\}$$

It has the first order condition

$$\beta \frac{\hat{R} - R}{(1 - I)R + \hat{R}I} + (1 - \beta) \frac{R_H + \hat{R} - 2R}{(1 - I)2R + (R_H + \hat{R})I} = 0,$$

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which leads to the following optimal risky investment.

$$I^* = \frac{R}{2}\left(\frac{1}{R - R_H} + \frac{1}{R - \hat{R}}\right) \tag{8}$$

As cash-in-the-market pricing happens at $t = 1$, market clearing requires the nominal value of assets on sale should be equal to the total cash available.

$$(1 - I)(1 - \beta) + \left(1 - \beta\right)\frac{I}{2}\hat{P} = \hat{P}\left[\beta I + (1 - \beta)\frac{I}{2}\right],$$

or in net terms, $(1 - \beta)(1 - I) = \beta PI$. With $\hat{P} = \hat{R}/R$, it gives another expression of the equilibrium investment.

$$I^* = \frac{1 - \beta}{\beta\hat{R}/R + (1 - \beta)}. \tag{9}$$

Equalizing expression (8) and (9), one can solve for the equilibrium returns of cash R.

$$R = \hat{R}\frac{(1 - \beta^2)R_H + (1 + \beta^2)\hat{R}}{(\beta^2 - \beta + 2)\hat{R} + (1 - \beta)\beta R_H} > 1$$

It is straightforward to verify that the cash-in-the-market pricing does create positive net returns for cash at $t = 1$. Most importantly, the resulting asset price for the mixed quality is smaller than 1.

$$\hat{P} = \frac{\hat{R}}{R} = \frac{(\beta^2 - \beta + 2)\hat{R} + (1 - \beta)\beta R_H}{(1 - \beta^2)R_H + (1 + \beta^2)\hat{R}} < 1$$

Therefore, the private information on asset qualities impairs the liquidity provision of market exchange. To be more precise, $C_1^* < 1$ for $\hat{P} < 1$, so that $t = 1$ consumption is distorted downward as compared to the first best level. Furthermore, one can show the patient consumers benefit from trading on private information at the cost of impatient consumers, in particular, $C_2^* > \hat{R}$, and $t = 2$ consumption shows an upward distortion as compared to the first-best level.

This information friction highlights the potential of the systemic asset to improve the allocation. The low asset price also limits investors' debt capacity. Because in early liquidation an impatient investor cannot recoup his initial investment and has to pay the liquidation cost out of his initial endowment, there is a cost of leverage. As leverage amplifies both $t = 2$ returns and $t = 1$ liquidation losses, an investors will restrict his leverage in light of the potential cost. This contrasts the classic Diamond-Dybvig model where the asset price is always equal to 1 so that the investors face no liquidation cost and the optimal leverage is not well defined.

Interestingly, the downward-distorted asset price ($\hat{P} < 1$) also implies that the amount of risky investment will exceed the first-best level.

$$I = \frac{1 - \beta}{\beta \hat{P} + (1 - \beta)} > 1 - \beta$$

This is because the adverse selection, while driving down asset prices and weakening the liquidity insurance, provides extra insurance on credit qualities. The patient investors are protected against the low returns R_L, because with the private information asset qualities they can sell their low returns assets in the secondary market.

Proposition 4. *The allocation under adverse selection features a unique price $\hat{P} < 1$ for the mixed asset quality, so that the asset liquidation value is smaller than the initial investment. The investors over-invest in the risky asset with $I^* > 1 - \beta$, leading to a downward distortion to $t = 1$ consumption, $C_1^* < 1$, and an upward distortion to $t = 2$ consumption, $C_2^* > \overline{R}$.*

Proof. See Appendix A.10. □

4.3   Trade-off: systemic risk vs. liquidity creation

In order to highlight the impacts of aggregate risk exposure and information asymmetry, I tabulate the main results for a side-by-side comparison.

Table 1: Equilibrium: frictionless, aggregate risk, & adverse selection

	Frictionless Market	Aggregate Exposure	Adverse Selection
$t = 0$ investment	$1 - \beta$	$1 - \beta$	$\frac{1-\beta}{\beta\hat{P}+(1-\beta)} > 1 - \beta$
$t = 1$ cash returns (price system)	$\overline{R}$	$wR_H + (1 - w)\overline{R}$ in state G $wR_L + (1 - w)\overline{R}$ in state B	$\frac{2\beta^2\hat{R}+(1-\beta^2)(R_H+\hat{R})}{(\beta^2-\beta+2)\hat{R}+(1-\beta)\beta R_H}\hat{R}$
$t = 1$ consumption (liquidation value)	1	1	$(1 - \hat{I}) + \hat{I}\hat{P} < 1$
$t = 2$ consumption	$\overline{R}$	$wR_H + (1 - w)\overline{R}$ in state G $wR_L + (1 - w)\overline{R}$ in state B	$(1 - \hat{I})R + \frac{R_H+\hat{R}}{2}\hat{I} > \overline{R}$

The key differences lie in $t = 1$ and $t = 2$ consumption. With aggregate risk exposure, market exchange can still provide sufficient liquidity insurance and implements the first-best $t = 1$ consumption, $c_1 = 1$. However, the investors will have to bear volatile consumption at $t = 2$. Under asymmetric information, the asset price is distorted downwards, $\hat{P} < 1$, resulting in weak liquidity insurance and low $t = 1$ consumption. But with idiosyncratic credit risk perfectly diversified, the investors will not have volatile $t = 2$ consumption. The comparison presents a conflict between systemic risk exposure and liquidity insurance. While the systemic asset limits private information and

preserves market liquidity, the idiosyncratic assets allows for better diversification. By setting the relative weights on the two types of assets, the investors choose their exposure to the aggregate risk and trade off between systemic risk and liquidity creation.

It should be emphasized that both cases are hypothetical because of the exogeneity of w. In the first case, once the aggregate exposure w is endogenous, the exposure will be optimally set at zero—a result we see in section 3.2. In the second case, the investors will voluntarily choose a positive w in order to better insure themselves against liquidity risk—the main result of the next section.

5 Full-fledged model

Having understood the cost and benefit of aggregate risk exposure, we are now ready to endogenize it. Once investors have private information on their asset qualities, the systemic asset is no longer dominated by the idiosyncratic ones. Because the correlated returns of the systemic asset limits private information and preserves market liquidity, the need for market liquidity will result in a positive exposure to the aggregate risk in equilibrium.

5.1 Endogenous aggregate risk exposure

As the investors will perfectly diversify the idiosyncratic credit risk, they face the following maximization program at $t = 0$. The analysis follows the same steps as in the simplified case. We show first that the private information on idiosyncratic assets leads to adverse selection. And the intermediate date returns of cash is positive, implying cash-in-the-market pricing. The proofs of the previous sections hold with proper modifications.

Lemma 10. *Under asymmetric information of asset qualities, the equilibrium price will be such that adverse selection happens. In addition, in both states B and G, cash yields positive returns at $t = 1$ and asset prices are determined by the limited cash supply.*

Proof. See appendix A.11. □

The investors solve the following $t = 0$ optimization program where the asset prices are such that the nominal value of assets on sale is equal to the cash available.

$$\max_{w,I} E\left[\beta\left(\frac{1}{2}\log c_{1,G} + \frac{1}{2}\log c_{1,B}\right) + (1-\beta)\left(\log c_{2,G} + \log c_{2,B}\right)\right]$$

$$s.t. \quad c_{1,B} = (1-I) + (1-w)I\hat{P}_B + wIP_L$$

$$c_{1,G} = (1-I) + (1-w)I\hat{P}_G + wIP_H$$

$$c_{2,B} = (1-I)R_B + (1-w)I(\hat{R} + R_H)/2 + wIR_L$$

$$c_{2,G} = (1-I)R_G + (1-w)I(\hat{R} + R_H)/2 + wIR_H$$

$$(\hat{R} + R_H)/2\hat{P}_B = R_L/P_L = R_B$$

$$(\hat{R} + R_H)/2\hat{P}_G = R_H/P_H = R_G$$

As the adverse selection problem reduces market liquidity, the investors will expose themselves voluntarily to the aggregate risk, because the systemic asset is now no longer dominated by the idiosyncratic. The high correlation in returns limits the scope of private information and promotes market liquidity. Sticking to $w = 0$ would be suboptimal because in that case the investors will not benefit from the liquidity insurance provided by the aggregate risk exposure.

Proposition 5. *Under asymmetric information of asset qualities, investors choose $w > 0$, exposing themselves voluntarily to the aggregate risk.*

Proof. See appendix A.12. □

The risk averse investors seek to smooth their consumption, requiring both diversification and market liquidity. Optimal portfolio choice balances between the two objectives. As emphasized by the paper, the asset of aggregate risk exposure limits private information on asset qualities by correlated asset returns. It has a strength in providing liquidity insurance but results in imperfect diversification. Whereas the idiosyncratic assets works exactly the other way around—allowing perfect diversification but providing poor liquidity insurance. More specifically, a diversified portfolio of idiosyncratic assets insures credit risk in two ways. First, the idiosyncratic credit risk is diversifiable so the returns are not affected by aggregate states. Second and less apparently, since the private information on idiosyncratic returns leads to adverse selection, the possibility to trade on private information also provides insurance for credit risk. By selling the 'lemons' in the market, the patient investors will not receive the low returns R_L and are insured against the worst scenario. Table 2 presents a comparison between the two types of assets.

Table 2: Insurance provision: the systemic vs. idiosyncratic assets

	Insurance for credit risk	Insurance for liquidity risk	Asset price volatile?
Idiosyncratic Assets	Yes, risk diversifiable	First-best liquidation value	Yes
Systemic Assets	No, aggregate risk	Poor liquidity insurance	No

5.2   Comparative statics

The result in the last section is qualitative—showing the exposure will be strictly positives. To quantify the results, I examine the comparative statics. The key question I try to answer is how the systemic exposure will change when the overall riskiness of the assets drops. In particular, I consider the case of mean-preserving contraction, where the high and low returns converge but the mean keeps unchanged. Since it is difficult to obtain a closed form solution, I report the numerical results in this section.

Figure 2 depict how w^* changes when R_L and R_H converge. The horizontal axis measures the magnitude of the contraction, with $\epsilon > 0$ added to R_L but subtracted from R_H. As ϵ increases, the gap between R_L and R_H closes.[17] Note that the impact on w^* is unclear priori. Intuitively, when the returns of R_H and R_L converge, diversification generates less benefit; and liquidity insurance becomes a more important concern. The change would result in an increasing w. But on the other hand, the convergence also reduces the severity of adverse selection and brings $t = 1$ asset price for idiosyncratic assets close to one. The numerical result shows that for reasonable parameters, the first effect dominates, and the aggregate risk exposure w^* slightly increases when R_L and R_H converge. The analysis provides necessary preparation for the discussion in section 6.1, where I study how an increase in the aggregate risk exposure affects investors' leverage.

6   Interaction with leverage

The intermediate date asset price naturally links to the collateral value, which plays a crucial role in determining a financial institution's debt capacity. Therefore I study how the aggregate risk exposure interacts with investors' leverage decisions.

For the investment is completely scalable, it still holds that investors will perfectly diversify the idiosyncratic risk. So I analyze directly the simplified optimization. The possibility to borrow slightly

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[17]All the restrictions on the parameters remain satisfied.

Figure 2: Comparative statics: mean-preserving contraction
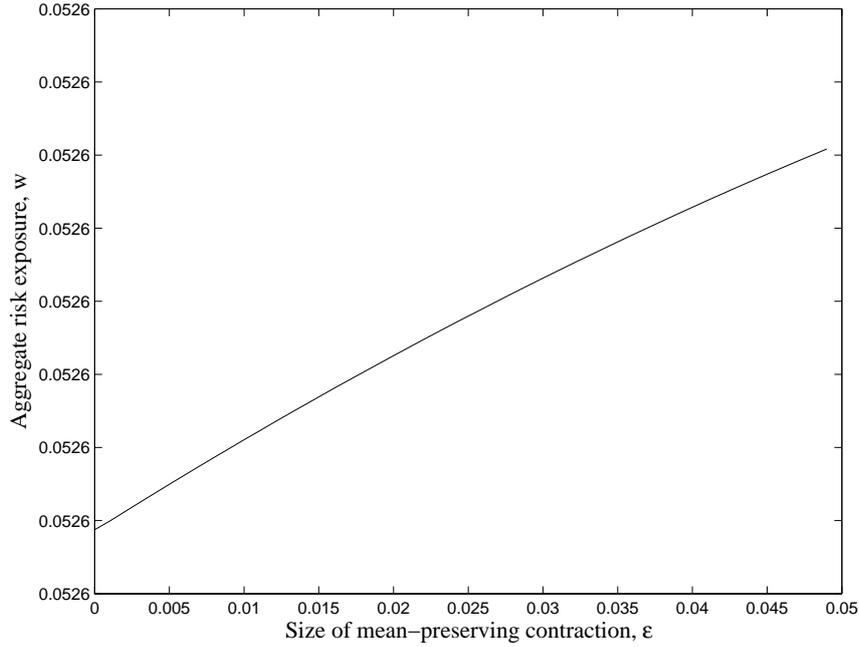
The figures are based on the following parameters: The probability of liquidity shock is one half, $\beta = 1/2$. The mean of expected returns is kept at $= 1.05$, with R_L increasing from 0.90 to 0.94 and R_H decreasing from 1.20 to 1.16.

change the optimization program.

$$\max_{w,I,D} E\left[\beta\left[\frac{1}{2}\log c_{1,G} + \frac{1}{2}\log c_{1,B}\right] + (1-\beta)\left[\log c_{2,G} + \log c_{2,B}\right]\right]$$

$s.t.$ $c_{1,B} = (D+1)[(1-I) + (1-w)I\hat{P}_B + wIP_L] - Dr$

$c_{1,G} = (D+1)[(1-I) + (1-w)I\hat{P}_G + wIP_H] - Dr$

$c_{2,B} = (D+1)[(1-I)R_B + (1-w)I(\hat{R} + R_H)/2 + wIR_L] - Dr$

$c_{2,G} = (D+1)[(1-I)R_G + (1-w)I(\hat{R} + R_H)/2 + wIR_H] - Dr$

$(\hat{R} + R_H)/2\hat{P}_B = R_L/P_L = R_B$

$(\hat{R} + R_H)/2\hat{P}_G = R_H/P_H = R_G$

It involves five endogenous variables, portfolio choice, I and w, price systems R_G and R_B, and the leverage decision D, and can be solved by a system of five equations: the three first order conditions with respect to w, I and D, as well as the two market clearing conditions. For closed form solutions are difficult to obtain, numerical results are reported in this section. I study how aggregate risk exposure and leverage mutually reinforce each other: (1) The aggregate risk exposure leads to greater

debt capacity; and (2) low borrowing cost and high leverage result in more aggregate risk exposure. The leverage and systemic risk therefore tend to go hand in hand.

6.1   Leverage driven by herding

A greater aggregate risk exposure would contribute to higher leverage. Intuitively, the leverage ratio is set to balance consumption across the two preference states: By using leverage, an investor will reduce his $t = 1$ consumption if he is hit by the liquidity shock, because the liquidation cost has to be paid by the initial endowment. But he will multiply his $t = 2$ consumption if he turns out to be patient. With a greater exposure to the systemic asset, the $t = 1$ liquidation cost declines, reducing the cost of leverage and making it more attractive.

I consider again the variance of returns as the driving force of portfolio adjustment. Figure 3 depicts the changes of the key endogenous variables when there is an increase in the aggregate risk exposure. The first observation is that once we allow for leverage, systemic asset becomes more attractive. For the same set of parameters, the optimal w is four times greater than in the case where the possibility of leverage is excluded (comparing Figure 2 and Figure 3a). Second, as the exposure to the aggregate risk increases, the reduced adverse selection leads to higher liquidation values (Figure 3b). And finally, as the aggregate risk exposure increases market liquidity, investors will use greater leverage because they will suffer smaller liquidation in the case of facing liquidity shocks (Figure 3c).

Proposition 6. *As systemic risk rises, investors reduces the cost of early liquidation and increase their leverage.*

6.2   Herding driven by leverage

The ease of borrowing also contributes to the build-up of systemic risk. In particular, I examine the impact of low risk-free rate. Figure 3 depicts how the endogenous variable respond to the change in the risk-free rate: When the risk-free rate declines, we will have in equilibrium a higher leverage, greater exposure to the systemic asset, and more volatile asset prices.

Proposition 7. *When risk-free rate r declines, investors increase their investment in the systemic asset, resulting in smaller liquidation losses and enabling them to use higher leverage.*

When the cost of funding declines, investors will be induced to borrow, in hope that the leverage will multiply their $t = 2$ consumption. But if an investor happens to be hit by a liquidity shock, he will have to sell his portfolio in the secondary market at a discount and bear the early liquidation loss with his initial endowment. As the early liquidation loss declines with the systemic exposure, investors will optimally reshuffle their portfolio to put more weights on the systemic asset.

Figure 3: Leverage Driven by Systemic Risk Exposure
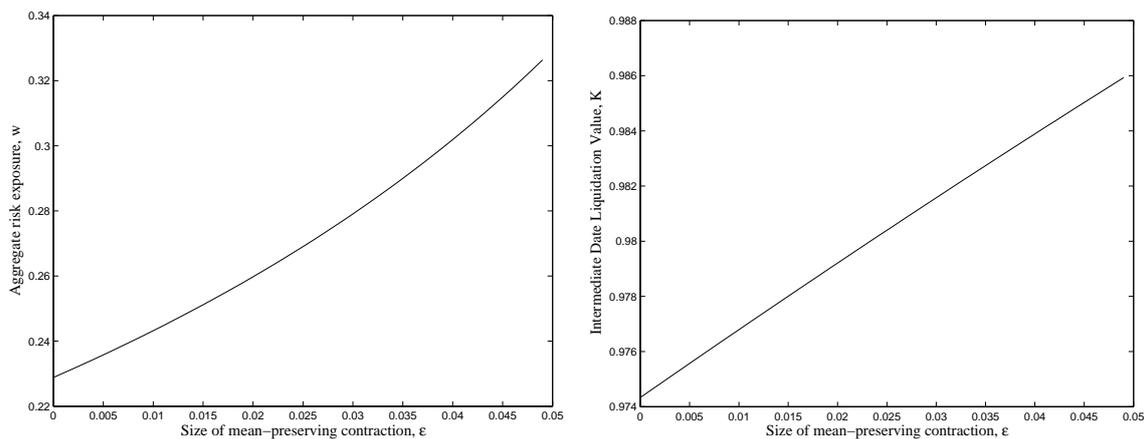
(a) Aggregate Risk Exposure

(b) Liquidation Value
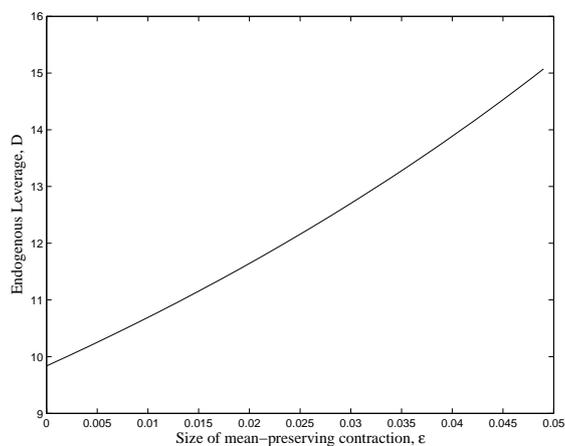
(c) Leverage

Following figure 2, the parameters are as follows: β is set as $1/2$. The mean of $\tilde{R}$ is kept at $= 1.05$, with R_L increasing from 0.90 to 0.94 and R_H decreasing from 1.20 to 1.16. The risk-free rate r is set at 1.01. The the changes in portfolio choices is still driven by the disturbance on the returns distribution.
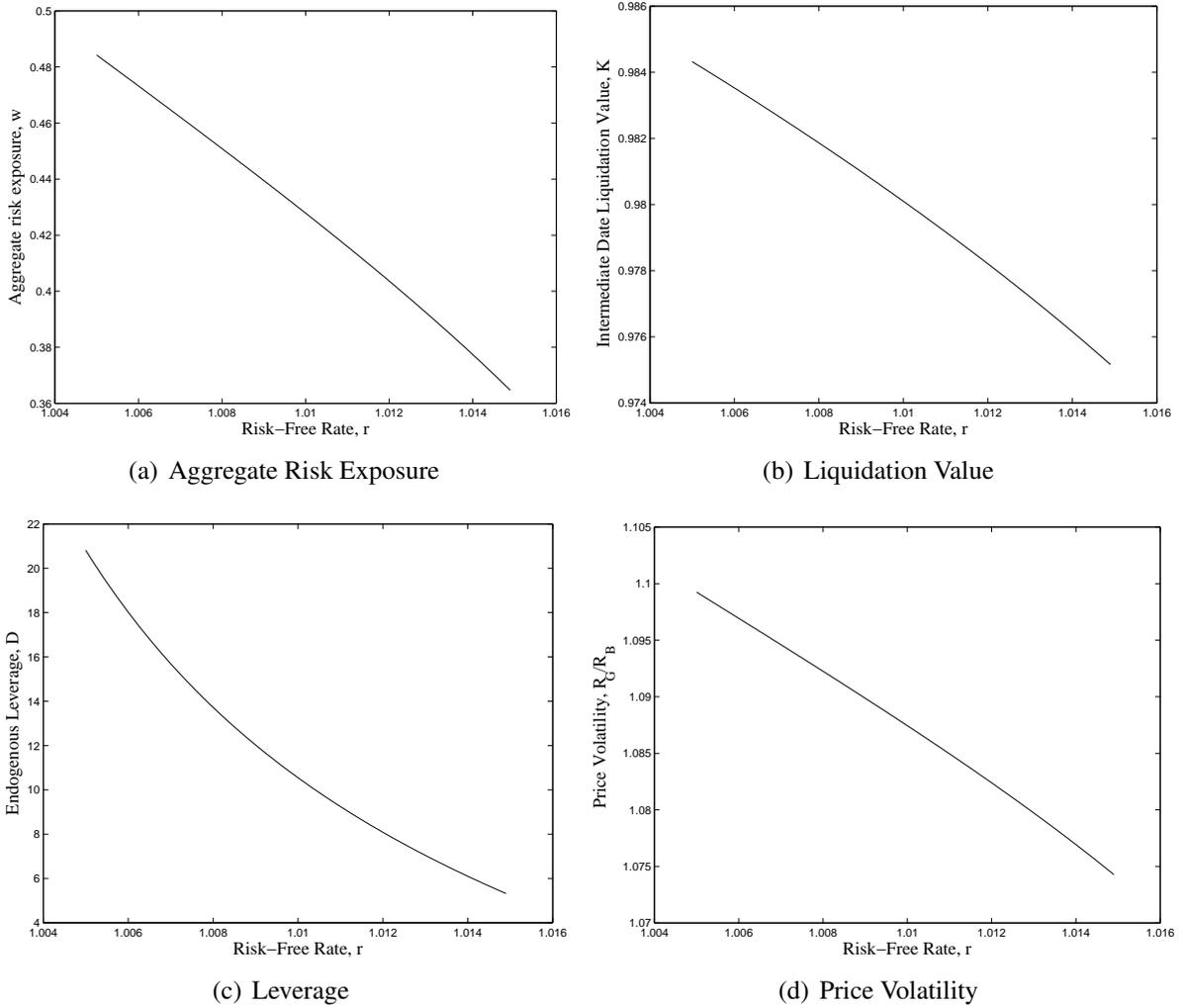
Figure 4: The Impact of Accommodative Credit Conditions

(a) Aggregate Risk Exposure

(b) Liquidation Value

(c) Leverage

(d) Price Volatility

The depicted comparative statics shows the impact of monetary policy. The horizontal axis denotes the net risk-free rate, climbing from 0.5% to 1.5%. $\beta = 1/2$ and $\overline{R} = 1.05$ as in the preceding figures, with $R_L = 0.90, R_H = 1.20$.

Greater systemic exposure also leads to higher asset price volatility because it ties $t = 1$ asset markets more tightly to the non-diversifiable aggregate risk. I use the ratio R_G/R_B to measure price volatility. When there is no systemic risk exposure, R_G and R_B are equal and the ratio takes the value of one. Otherwise, the returns of cash are higher in state G than in state B, implying $R_G/R_B > 1$. The higher the ratio is, the more volatile the asset prices are across aggregate states.

7 Empirical implications

When the asset of systemic risk is interpreted as mortgage assets, the stylized model makes two testable hypotheses about banks' exposures to housing market: First, when a bank increases its leverage using more short-term debt, it will reshuffle its asset composition, putting a greater weight

on the systemic but liquid mortgage assets. And this allows the bank to cope with the increasing rollover risk. Second, when the cost of short-term funding becomes low, banks will find it attractive to borrow more. And this will again trigger the first channel and leads to a greater portfolio weight on the mortgage assets.

1. A bank's exposure to mortgage assets *increases* in its use of short-term funding.

2. A bank's exposure to mortgage assets *increases* in the ease of borrowing.

For evidence on these two predictions, I study how banks' holdings of mortgage-backed securities depend on their use of short-term funding as well as the ease of borrowing in money market.[18] Using call report data between 2002Q1 and 2006Q2, I estimate the following regression model for the 400 biggest bank holding companies in the U.S..

$$Mortgage_{it} = \beta_0 + \beta_1 BankCharacter_{it-1} + \beta_2 MacroVar_{t-1} + \gamma_1 FundCost_{t-1} + \gamma_2 StFund_{it-1} + \alpha_i + \epsilon_{it}$$

I measure the exposure to mortgage assets by the fraction of mortgage-backed securities in a bank's total asset, because MBS are ready for sale and are most close to the theoretical setup. As a starting point, I use fraction of repo funding to proxy the short-term borrowing "$StFund$". Accordingly, the repo rate is used to proxy the cost of short-term debt "$FundCost$". Admittedly the measurement has its limitation: The repo rate can only be positive may give an incomplete description of the ease of borrowing. For this reason, I also use one alternative measurement—the deviation of effective federal fund rate from the rate suggested by Taylos's rule, because accommodative monetary policies eases borrowing. Based on the theoretical prediction, one would expect positive signs for γ_1 and γ_2. In addition to the two main variables, I control for bank characteristics *BankCharacter* and macroeconomic conditions *MacroVar*. Furthermore, to take into account any time-invariant bank characteristics that can be correlated with the regressors, I allow for firm fixed-effect (α_i). A complete list of variable definitions and summary statistics can be found in Appendix B.

Table 3 shows the main findings and lends preliminary support to the theoretical model. The main results are reported in column (1) and (4). In both cases, the use of short-term funding (repo) is found to be positively correlated with banks' exposures to mortgage-backed securities. Low repo rate (column (1)) and accommodative monetary policy (as in column (4)) are also found to be positively correlated with banks' holding of mortgage-backed securities.

Because the low interest rate affects banks' exposure to mortgage-backed securities via short-term debts, I conjecture that the effect of low interest rate is stronger for banks that use more wholesale short-term funding. To shed some light on this, the same analysis is done for two sub-samples: banks whose use of wholesale short-term funding is below the sample median (column (2) and (5))

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[18]Alternatively, the systemic risk can also be captured by market-based systemic risk measurements such as Marginal Expected Shortfall (MES) or CoVaR.

Table 3: How do banks' exposures to MBS change with short-term debt and its cost?

The regressions investigate how banks' exposures to mortgage-backed securities (as a fraction of total assets) change with their use of wholesale short-term funding (measured by repo funding as a fraction of total liabilities) and credit conditions (measured by repo rate and deviation of federal fund rate from Taylor's rule). To mitigate potential endogeneity, explanatory variables are all lagged by one period. All specifications include bank fixed-effect to control for unobserved bank characteristics. The standard errors are clustered on individual bank level to take into account potential auto-correlation in the errors.

	(1) MBS/Asset	(2) MBS/Asset	(3) MBS/Asset	(4) MBS/Asset	(5) MBS/Asset	(6) MBS/Asset
Bank Size	-1.036	-0.084	-2.568*	-1.374	-0.403	-3.060**
	(0.903)	(0.867)	(1.365)	(0.945)	(0.912)	(1.410)
Returns on Assets	0.372***	0.312**	0.489***	0.300**	0.243*	0.393**
	(0.122)	(0.144)	(0.168)	(0.116)	(0.130)	(0.163)
Risk Weighted Asset/Asset	-0.229***	-0.240***	-0.217***	-0.230***	-0.241***	-0.219***
	(0.026)	(0.031)	(0.041)	(0.026)	(0.031)	(0.041)
Total Asset/Equity	0.246**	0.239**	0.229	0.243**	0.238**	0.223
	(0.100)	(0.116)	(0.146)	(0.099)	(0.115)	(0.145)
GDP Growth	-0.040	-0.079	0.007	-0.138	-0.159	-0.136
	(0.113)	(0.147)	(0.151)	(0.089)	(0.124)	(0.123)
Unemployment Rate	-0.143	-0.389	-0.459	0.843**	0.562*	0.709
	(0.438)	(0.471)	(0.635)	(0.329)	(0.327)	(0.494)
Repo Funding/Liabilities	0.199***	0.085	0.197***	0.201***	0.081	0.200***
	(0.066)	(0.102)	(0.071)	(0.067)	(0.102)	(0.071)
Repo Rate	-0.389***	-0.377***	-0.470***			
	(0.101)	(0.129)	(0.145)			
Deviation from Taylor's Rule				-0.177***	-0.161*	-0.238***
				(0.066)	(0.086)	(0.091)
N	6266	2942	3324	6266	2942	3324
adj. R^2	0.212	0.198	0.217	0.211	0.197	0.217

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

and those above the sample median (column (3) and (6)). The results are confirmative: For banks' exposure to mortgage-backed assets, the impact of low repo rate or accommodative monetary policy is stronger for banks that use more wholesale funding.

The preliminary results add to empirical studies on monetary policy and risk-taking, which have focused on individual banks. Representative of the approach are Jiménez et al. (2009), Ioannidou et al. (2009) and Dell'Ariccia et al. (2011). The authors use loan level data to study if the lending standards deteriorate under loose monetary policy. The current paper presents a different perspective: The ease of short-term borrowing can also affect banks' portfolio compositions, with accommodative monetary policies increasing banks' exposure to systemic risk.

8   Discussion and extensions

In this section, I discuss two main approaches of studying market liquidity in the existing literature, especially how they conceptually differ from each other and generate different implications. I also discuss some extension of the paper, how it can be rephrased with alternative modeling framework and how it contributes to policy issues such as formulating liquidity requirements.

8.1   Illiquidity: agency cost vs cash-in-the-market

In its broad sense, market liquidity refers to an asset's ability to be transformed into other assets without much loss of its fundamental value. The seemingly clear definition, however, allows important nuances in defining the source and creation of market liquidity. One strand of literature explains market illiquidity by agency costs. For example, Shleifer and Vishny (1992) ties illiquidity to inalienable human capital (moral hazard) and the others like Dang et al. (2009) studies the impact of information asymmetry (adverse selection).[19] The other strand of the literature, represented by Allen and Gale (1994), takes a cash-in-the-market approach and consider limited short-term cash supply as a driving force for market illiquidity. The two approaches lead to different concepts of liquidity. While in the cash-in-the-market approach, liquidity is narrowly defined as 'cash', the agency cost based model considers 'cash' to be liquid because it involves neither moral hazard or adverse selection, implying minimum agency cost in transactions.

The conceptual differences extend to implications. To see the point, consider a preference shock that makes more investors want to consume early. In cash-in-the-market type of models, this implies a greater supply of assets, an aggregate shortage of cash, and therefore low asset prices and liquidity. But for the agency cost-based approach, such a preference shock means asset sales more likely due to

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[19]The leading example of creating liquidity by limiting moral hazard should include the equity tranche in securitization and lead bank in syndicated loans.

liquidity reasons, resulting in better average asset qualities, greater willingness to pay for the assets, and therefore higher market liquidity.

Since adverse selection and limited short-term cash supply co-exist in the current model, it may help to clarify the confusing discrepancy. In particular, the model suggests that the two approaches examine different aspects of illiquidity, with the key difference rooted in the presence of aggregate uncertainty. The cash-in-the-market illiquidity relies crucially on the existence of aggregate uncertainty. As a result, the illiquidity takes the form of price volatility. On the other hand, the agency cost-based illiquidity can exist without aggregate uncertainty. The illiquidity is caused by market friction such as moral hazard or adverse selection. By and large, I consider the cash-in-the-market approach captures illiquidity via the price volatility, while the agency cost approach captures illiquidity via the low liquidation values.

8.2 Exogenous vs endogenous liquidity risk

The idea that an asset of aggregate risk can promote market liquidity is general and not dependent of the specific modeling approach of the current paper. In particular, one can relax the assumption of exogenous liquidity shocks and model liquidity risk arising from weak fundamentals. For instance, one can write a model where investors are risk neutral and idiosyncratic assets are attractive because they allow good diversification that reduces bankruptcy probabilities. Yet as argued by the current paper, the evaluation of idiosyncratic assets often relies on soft and private information. This would lead to adverse selection and illiquidity. Therefore, a potential fire sale loss is related to the idiosyncratic assets and can make creditors panic and run.[20] In this sense, idiosyncratic assets reduce pure insolvency risk by its diversification, but increases the risk of bank runs because of illiquidity. By contrast, an asset of aggregate risk reduces fire sale losses and bank run risk by limiting private information and adverse selection, but it contributes to the insolvency risk because the risk is not diversifiable. Therefore, when facing a trade-off between pure insolvency and illiquidity risk, investors will still voluntarily expose themselves to the asset of aggregate risk in order to minimize their total bankruptcy probabilities.

8.3 Liquidity requirement

To understand the nature of illiquidity has important policy implications. For the last banking crisis is to a large extent a liquidity crisis, a direct policy response is to impose liquidity requirements.[21]

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[20]Runs caused by fire sale losses are common feature in the fundamentals-based bank run literature as exemplified by papers such as Morris and Shin (2000), Rochet and Vives (2004), and Goldstein and Pauzner (2005). And using also the global games approach, Li and Ma (2012) present a model where fire sale losses are based on adverse selection.

[21]For example, Basel III proposes 'liquidity coverage' ratio and 'net stable funding ratio'.

To formulate such regulation, a natural question is what assets are qualified as 'liquid assets'. Shall the regulation allows only cash and cash alike assets (such as treasury bills)? Or should it take a more general approach and allow also assets that involves relatively little agency cost in transaction? To understand the nature of liquidity and how it is created will provide a theoretical ground for these questions. One policy move is especially interesting in this respect. In January 2013, regulators softened the original Basel III rules and expanded the definition of liquid assets. In particular, now a portion of the liquid assets can be made up of mortgage-backed securities. While many may consider this as a victory of banks' lobbying, the current paper provides a possible justification for this move. The more general definition of liquid assets. In the author's opinion, a very narrowly defined liquidity regulation can not necessarily protect banks from the risk of runs and may even backfire. While cash alike assets do not incur liquidation costs upon runs, their risk-free nature usually result in very low returns. It impairs banks' profitability and makes banks less able to build up capital buffers to absorb fire-sale losses on the other parts of the portfolio. As a result, the run can be accelerated. In this respect, assets such as mortgage-backed securities are quite special for liquidity regulation. On the one hand, such assets can involve relatively little agency cost and therefore more liquid than assets such as relationship loans. On the other hand, unlike cash, mortgage-backed securities generate positive returns and enables bank to build up a buffer against fire sale losses. Therefore rather than a compromise, that policy move can be in the direction of optimal regulation.

8.4   A few remarks on the caveats

Before concluding, I would like to point out a few of caveats of the paper. First, the model is not able to generate the coincidence of systemic exposure to mortgage assets and market freeze-up—an extreme form of illiquidity that featured the recent crisis. The illiquidity in the model takes only a 'moderate' form of price volatility, because the impatient investors always sell in the market, preventing the market from collapsing. The second and related issue is the use of log utility function. While it excludes the distortion due to contract incompleteness and helps isolate the impact of adverse selection, it also leads to the result of no endogenous bankruptcy. The absence of bankruptcy risk clearly presents a limit on the model's interpretation in the context of financial stability. Third, the assumption of risk aversion leaves a gap between the model and the real world financial institutions, for those are usually conceived to be well diversified and risk neutral. While all the three limitations relate to the modeling choice of using Diamond-Dybvig framework, as I point out in section 8.2, the notion that correlated returns limit private information and promotes market liquidity is not dependent of the framework. The generalization of the notion, as well as its implication for public policies, would invite future research.

9 Conclusion

The paper develops a model where investors' herding behaviors are driven by their need for market liquidity. Based on the notion that market liquidity relies on information symmetry, the paper emphasizes that an asset of systemic risk can limit the scope of private information and thereby fosters market liquidity creation. Systemic risk taking emerges as a part of a second-best allocation where investors face liquidity risk and informational constraints. Because such liquidity creation only comes at the cost of potential systemic crises, the paper suggests a basic trade-off between systemic risk and private liquidity creation. To the extent that the performance of mortgage assets is tied to the systemic risk of house price, the paper offers an explanation of why financial institutions systemically exposed themselves to mortgages and mortgage-backed securities in the recent crisis.

The theoretical framework also allows us to investigate the interaction between leverage and systemic risk taking. Consistent with empirical observations, the stylized model predicts that financial institutions leveraged with more wholesale short-term funding tend to hold more liquid but systemic assets. Moreover, the ease of borrowing contributes to the build-up of systemic risk. Given the prolonged low-interest-rate period prior to the sub-prime crisis, the model helps to explain why banks' leverage and their systemic exposures rose hand in hand under accommodative credit conditions.

Appendix A   Proof of propositions

Appendix A.1   Proof of lemma 1

The proof makes use of the argument of second order stochastic dominance. Consider F the distribution of the returns generated by a portfolio of idiosyncratic assets that all have equal weights $v_i = 1/N$. Since the assets are *i.i.d.*, any deviation from the equal-weight portfolio will generate a mean-preserving spread of F and is second order stochastic dominated. Given the investors are risk averse, a portfolio with equal weight on every asset will be most preferred and the social planner chooses $v_i^* = 1/N$.

Appendix A.2   Proof of lemma 2

To verify that the social optimal aggregate risk exposure is zero, I show the expected utility monotonically decreases in w. The social planner faces the following unconstrained optimization.

$$\max_{w,I} \left\{ \frac{1}{2} \left[\beta \log(1-I) + (1-\beta) \log \left(\frac{wR_L + (1-w)\overline{R}}{1-\beta} I \right) \right] + \frac{1}{2} \left[\beta \log(1-I) + (1-\beta) \log \left(\frac{wR_H + (1-w)\overline{R}}{1-\beta} I \right) \right] \right\}$$

It has the following first order derivative with respect to w.

$$\frac{1-\beta}{2}\left(\frac{R_L - \overline{R}}{wR_L + (1-w)\overline{R}} + \frac{R_H - \overline{R}}{wR_H + (1-w)\overline{R}}\right)$$

Notice that the second order condition is satisfied and the expression monotonically decreases in w. The expression is equal to zero when $w = 0$. Therefore, the first order derivative is less or equal to zero on the whole range of w, and the social planner will choose zero exposure to the aggregate risk, $w^* = 0$.

Appendix A.3 Proof of proposition 1

The result follows directly lemma 1 and 2. As the social planner optimally chooses $w = 0$ and perfectly diversifies the idiosyncratic credit risk, his optimization program (3) reduces to the classic Diamond-Dybvig case. The unconstrained optimization program

$$\max_I \left\{\beta \log\left(\frac{1-I}{\beta}\right) + (1-\beta)\log\left(\frac{I\overline{R}}{1-\beta}\right)\right\}$$

yields optimal investment $I^* = 1 - \beta$, and the corresponding consumption $c_1^* = 1$, $c_2^* = \overline{R}$.

Appendix A.4 Proof of lemma 4

I prove by contradiction that asset prices cannot be equal to fundamentals in both state B and G. Suppose that the opposite is true: $P_H^s = P_G = R_H$, $P_L^s = P_B = R_L$, and the gross returns of cash, R_B and R_G, are equal to 1. The investors' maximization $t = 0$ program (4) will reduce to the following form.

$$\max_{w,I} \left\{\frac{1}{2}\log\left[(1-I) + wIR_H + (1-w)I\overline{R}\right] + \frac{1}{2}\log\left[(1-I) + wIR_L + (1-w)I\overline{R}\right]\right\}$$

Examine the first order derivative with respect to w.

$$\frac{IR_H - I\overline{R}}{(1-I) + I\overline{R} + (R_H - \overline{R})wI} + \frac{IR_L - I\overline{R}}{(1-I) + I\overline{R} + (R_L - \overline{R})wI}$$

$$= \frac{2(R_H - \overline{R})(R_L - \overline{R})wI}{[(1-I) + I\overline{R} + (R_H - \overline{R})wI][(1-I) + I\overline{R} + (R_L - \overline{R})wI]}$$

Given $R_H > \overline{R}$ and $R_L < \overline{R}$, the expression is equal to or less than zero independent of w and I, and reaches zero only when $w = 0$. So the optimal w is zero, and the optimization further reduces.

$$\max_I \left\{\log\left[(1-I) + I\overline{R}\right]\right\}$$

The objective function monotonically increases in I, implying $I^* = 1$. But this contradicts the existence of the secondary market.

Appendix A.5   Proof of lemma 6

I show that if cash-in-the-market pricing only happens in state G, the optimal choice of w is zero, so that such asymmetric equilibrium cannot exist. Suppose that asset prices are equal to fundamentals in state B. So we have $P_L = R_L$, and the gross returns of cash at $t = 1$ are equal to 1, $R_B = 1$. The maximization program (4) reduces to the following.

$$
\max_{w,I} \left\{
\begin{array}{l}
\frac{1}{2} \log\left[(1 - I) + wIR_L + (1 - w)I\overline{R}\right] + \\
\frac{\beta}{2} \log\left[(1 - I) + wI\frac{R_H}{R_G} + (1 - w)I\frac{\overline{R}}{R_G}\right] + \frac{1-\beta}{2} \log\left[(1 - I)R_G + wIR_H + (1 - w)I\overline{R}\right]
\end{array}
\right\}
$$

Its first order derivative with respect to w is

$$
\frac{IR_L - I\overline{R}}{(1 - I) + I\overline{R} + (R_L - \overline{R})wI} + \frac{IR_H - I\overline{R}}{(1 - I)R_G + I\overline{R} + (R_H - \overline{R})wI}.
$$

Note that the second order condition with respect to w is satisfied. To prove $w^* = 0$, one only needs to show the expression is negative at $w = 0$. When $w = 0$, the expression becomes

$$
\frac{IR_L - I\overline{R}}{(1 - I) + I\overline{R}} + \frac{IR_H - I\overline{R}}{(1 - I)R_G + I\overline{R}} = \frac{I(R_L - \overline{R})[(1 - I)R_G + I\overline{R}] + (R_H - \overline{R})[(1 - I) + I\overline{R}]}{[(1 - I) + I\overline{R}][(1 - I)R_G + I\overline{R}]}
$$

Because the denominator is positive we focus on the numerator to tell whether this whether this expression is negative. Note that by the definition of $\overline{R}$, we have $\Delta \equiv (R_H - \overline{R}) = -(R_L - \overline{R})$. Therefore the numerator reduces to

$$
\Delta(1 - I)(1 - R_G)I
$$

Note that by the presumption that cash-in-the-market happens in state G, we know $R_G > 1$ and the expression above is negative. Therefore, the optimal $w^* = 0$ and the case reduces to the classic Diamond-Dybvig model where $R_B = R_G > 1$. This contradicts our presumption.

Appendix A.6   Proof of lemma 7

It takes two steps to see $w^* = 0$. First, I show the liquidation value is always equal to 1. Note that optimization (4) has the following first order condition with respect to I.

$$
\frac{K_B - 1}{1 + (K_B - 1)I} + \frac{K_G - 1}{1 + (K_G - 1)I} = 0
$$

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Furthermore, the two market clearing conditions (5) and (6) imply that $K_B = K_G$. So the liquidation value of the portfolio is independent of the aggregate states. This is because investment is made at $t = 0$, so at date-1 the cash available to clear the market is in fixed supply. Combined with the first order condition of I, we will have

$$K_B = K_G = 1.$$

Second, we verify $w^* = 0$ by showing that the expected utility decreases in w. Note that the first order derivative with respect to w is as follows.

$$\frac{(R_L - \overline{R})/R_B}{1 + (K_B - 1)I} + \frac{(R_H - \overline{R})/R_G}{1 + (K_G - 1)I}$$

For $K_B = K_G = 1$, the first order derivative further reduces to

$$\frac{R_L - \overline{R}}{R_B} + \frac{R_H - \overline{R}}{R_G}. \tag{A.10}$$

By the definition of K_B and K_G, we have

$$R_B = (1 - w)\overline{R} + wR_L \quad \text{and} \quad R_G = (1 - w)\overline{R} + wR_H.$$

Substitute R_B and R_G into (A.10). The first order derivative becomes

$$\frac{R_L - \overline{R}}{\overline{R} + (R_L - \overline{R})w} + \frac{R_H - \overline{R}}{\overline{R} + (R_H - \overline{R})w}.$$

Note that the expression is equal to zero when evaluated at $w = 0$. Since the second order condition is satisfied, the first order derivative monotonically decreases in w, and is less or equal to zero on the whole range of w, implying $w^* = 0$.

Appendix A.7   Proof of proposition 2

The program yields unconstrained optimization.

$$\max_I \left\{ \beta \log[(1 - I) + I\overline{R}/R] + (1 - \beta) \log[(1 - I)R + I\overline{R}] \right\}$$

As in the classic Diamond-Dybvig setup, there is no aggregate uncertainty. And the only possible equilibrium returns of cash are $R = \overline{R}$. When $R < \overline{R}$, the objective function monotonically increases in I, implying $I^* = 1$. When $R > \overline{R}$, the objective function monotonically decreases in I, suggesting $I^* = 0$. Once we substitute $R = \overline{R}$ into the constraints, all the other results in the proposition will follow.

Appendix A.8   Proof of lemma 8

I now prove by contradiction that a price $P \le R_L$ cannot rise in equilibrium. Suppose $P \le R_L$ and denote the intermediate date returns of cash by R, $R \equiv \hat{R}/P$. The investors' optimization program becomes

$$\max_I \left\{ \beta \log\left[(1-I) + IP\right] + (1-\beta) \log\left[(1-I)R + \overline{R}I\right] \right\}$$

It has the following first order derivative with respect to I.

$$\beta \frac{P-1}{(1-I)+PI} + (1-\beta)\frac{\overline{R}-R}{(1-I)R+\overline{R}I} = \frac{1}{R/(\overline{R}-R)+I}$$

As the second order condition satisfies, the expression monotonically decreases in I. For the optimal risky investment I to be positive, the expression must be positive at $I = 0$ and negative at $I = 1$. Yet at $I = 0$, the expression reduces to

$$\frac{\overline{R}-R}{R} = P - 1.$$

It has a negative value by the presumption $P < R_L < 1$. Thus $P < R_L$ cannot rise in equilibrium. Such a price is too low to induce any positive risky investment at $t = 0$.

Appendix A.9   Proof of lemma 9

I show that for asset price equal to $\hat{R}$ the investors' expected utility monotonically increases in I and they will choose the optimal investment amount $I^* = 1$.

Suppose $\hat{P} = \hat{R}$. Because $\hat{R} > R_L$, adverse selection happens. The maximization program becomes

$$\max_I \left\{ \beta \log\left[(1-I) + I\hat{R}\right] + (1-\beta) \log\left[(1-I) + \frac{R_H + \hat{R}}{2}I\right] \right\}$$

The first order derivative with respect to I is

$$\beta \frac{\hat{R}-1}{(1-I)+I\hat{R}} + (1-\beta)\frac{(R_H+\hat{R})/2 - 1}{(1-I)+(R_H+\hat{R})I/2} = \frac{\beta}{1/(\hat{R}-1)+I} + \frac{1-\beta}{2/(R_H+\hat{R}-2)+I}$$

Note that the second order condition is satisfied. For the investment amount has an interior solution, first order derivative must be costive when $I = 0$ and negative when $I = 1$. When evaluated at $I = 1$, the expression takes the following form.

$$\frac{\beta}{\hat{R}/(\hat{R}-1)} + \frac{1-\beta}{(R_H+\hat{R})/(R_H+\hat{R}-2)} = 1 - \beta\frac{1}{\hat{R}} - (1-\beta)\frac{1}{(R_H+\hat{R})/2} \tag{A.11}$$

If (A.11) is positive, we know that the expected utility monotonically increasing in I and $I^* = 1$. This is equivalent to prove

$$\hat{R}(R_H + \hat{R}) - \beta(R_H + \hat{R}) - (1 - \beta)2\hat{R} > 0. \tag{A.12}$$

or after some rearrange

$$(\hat{R} - 1)(R_H + \hat{R}) + (1 - \beta)(R_H - \hat{R}) > 0. \tag{A.13}$$

Note that $R_H - \hat{R} = (R_H - R_L)/(1 + \beta)$ and expression (A.13) monotonically increases in R_H. Furthermore, by assumption $R_L R_H \geq 1$, for a given R_L the minimum value of R_H is $1/R_L$. It is sufficient to prove that (A.13) is positive at $R_H = 1/R_L$. In that case, (A.13) becomes

$$\left(\frac{\beta + R_L^2}{(1 + \beta)R_L} - 1 \right) \left(\frac{1}{R_L} + \frac{\beta + R_L^2}{(1 + \beta)R_L} \right) + (1 - \beta) \frac{1 - R_L^2}{(1 + \beta)R_L}$$

$$= \frac{1 - R_L}{(1 + \beta)^2 R_L^2} \left[(\beta - R_L)(1 + 2\beta + R_L^2) + (1 - \beta^2)(1 + R_L)R_L \right].$$

The sign of the expression depends on the terms in square bracket, which can be written as

$$(R_L^2 - R_L^3) + \beta(R_L - 1)^2 + \beta^2(2 - R_L - R_L^2) > 0$$

for $R_L < 1$. We prove the first order derivative is positive at $I = 1$ and positive on the whole interval $[0, 1]$. Thus the optimal investment will be $I^* = 1$, and $\hat{P} = \hat{R}$ cannot be an equilibrium.

Appendix A.10   Proof of proposition 4

With $\hat{P}$, it is straightforward to see $C_1^* = (1 - \hat{I}) + \hat{I}\hat{P} < 1$. Now I prove $C_2^* > \overline{R}$. Recall that $\hat{I} = \frac{(1 - \beta)R}{\beta\hat{R} + (1 - \beta)R}$ and $1 - \hat{I} = \frac{\beta\hat{R}}{\beta\hat{R} + (1 - \beta)R}$.

$$\begin{aligned} C_2^* &= (1 - \hat{I})R + \frac{R_H + \hat{R}}{2}\hat{I} \\ &= \frac{1}{\beta\hat{R} + (1 - \beta)R} \left[\beta\hat{R}R + \frac{R_H + \hat{R}}{2}(1 - \beta)R \right] \\ &= \frac{1}{\beta\hat{R} + (1 - \beta)R} \frac{R}{2} \left[(1 - \beta)R_H + (1 + \beta)\hat{R} \right] \end{aligned}$$

Substitute $\hat{R} = \frac{\beta R_H + R_L}{1 + \beta}$ into the expression above. We have

$$\begin{aligned} C_2^* &= \frac{R}{\beta\hat{R} + (1 - \beta)R} \frac{R_H + R_L}{2} \\ &= \frac{1}{\beta\frac{\hat{R}}{R} + (1 - \beta)} \overline{R}. \end{aligned}$$

Note that $\frac{\hat{R}}{R} = \hat{P} < 1$. Therefore, we show $C_2^* > \overline{R}$.

Appendix A.11   Proof of lemma 10

Step 1: No adverse selection in both state B and G cannot be an equilibrium.

Suppose that the opposite is true, so that $\hat{P}_B < R_L$ and $\hat{P}_G < R_L$. Here $\hat{P}_B$ and $\hat{P}_G$ are the asset price for mixed quality $\overline{R}$ in state B and G respectively. The maximization program takes the form of program (4). I show the first order derivative with respect I

$$\frac{1}{2}\left[\frac{wR_L + (1-w)\overline{R} - R_B}{(1-I)R_B + wIR_L + (1-w)I\overline{R}} + \frac{wR_H + (1-w)\overline{R} - R_G}{(1-I)R_G + wIR_H + (1-w)I\overline{R}}\right]$$

is always negative. Evaluate the expression at $I = 0$, we will have

$$\left(\frac{\overline{R}}{R_B} + \frac{\overline{R}}{R_G} - 2\right) + w\left(\frac{\Delta}{R_G} - \frac{\Delta}{R_B}\right). \tag{A.14}$$

By presumption that $\hat{P}_B = \overline{R}/R_B < R_L < 1$ and $\hat{P}_G = \overline{R}/R_G < R_L < 1$. The first term of (A.14) is negative. To prove the whole expression, it is sufficient to show $R_G \geq R_B$. We have a few distinct cases depending on whether cash-in-the-market pricing happens. a) When asset prices equal to fundamentals in both states, we will have $R_B = R_G = 1$. b) When cash-in-the-market happens only in one state, it can only happen in state G. In this case $R_G > 1$ and $R_B = 1$. c) When cash-in-the-market happens in both states, market clearing conditions imply

$$w\frac{R_H}{R_G} + (1-w)\frac{\overline{R}}{R_G} = w\frac{R_L}{R_B} + (1-w)\frac{\overline{R}}{R_B} \tag{A.15}$$

and $R_G > R_B$. Therefore, if adverse selection happens in neither state, the asset price will be low enough such that $I^* = 0$. And this cannot be an equilibrium.

Step 2: Asset prices equal to fundamentals in both states cannot be an equilibrium.

Suppose that the opposite is true. We will have $R_B = R_G = 1$. a) If at the same time, adverse selection does not happen in either one of the two states, or both, we will have $R_L > \hat{R}/R_B$ or $R_L > \overline{R}/R_B$, or both. This leads to the contradiction $R_L > \overline{B}$. b) If adverse selection happens in both states, the price for asset will be $\hat{R}$. We know from Lemma 9 that the price will be so high that the optimal investment is I^*.

Step 3: Cash-in-the-market in state G but not in B and at the same time adverse selection in G but not in B cannot be an equilibrium. Similarly, cash-in-the-market in state B but not in G and at the same time adverse selection in B but not in G cannot be an equilibrium.

Starting with the first statement, suppose that the opposite is true. It is implied that $R_B = 1$, $R_L < \hat{R}/R_G$ and $R_L > \overline{R}/R_B$. And this leads to the contradiction $R_L > \overline{R}$. Similarly, the second statement requires $R_G = 1$, $R_L < \hat{R}/R_B$ and $R_L > \overline{R}/R_G$, which again leads to the contradiction $R_L > \overline{R}$.

Step 4: Cash-in-the-market in both states, and at the same time adverse selection in state G but not in state B, cannot be an equilibrium.

Suppose that the opposite is true. The market clearing requires (A.15). Rearrange terms, we will have

$$\frac{R_B}{R_G} = \frac{wR_L + (1-w)\overline{R}}{wR_H + (1-w)\hat{R}}.$$

At the same time the presumption about adverse selection requires $\overline{R}/R_B < R_L < \hat{R}/R_G$, or $R_B/R_G > \overline{R}/\hat{R}$. Combine the two conditions, we will need

$$\frac{wR_L + (1-w)\overline{R}}{wR_H + (1-w)\hat{R}} > \frac{\overline{R}}{\hat{R}}.$$

Note that the LHS decreases in w. And when $w = 0$, the LHS reaches its maximum $\overline{R}/\hat{R}$. Therefore, we derive the contradiction.

Appendix A.12 Proof of proposition 5

In order to show that the optimal w is greater than 0, I evaluate the first order derivative of w at $w = 0$ and show it is positive. With $(R_H + \hat{R})/2$ denoted by $\overline{R}'$, the first order derivative of w is

$$\frac{\beta}{2}\left[\frac{(-\hat{R} + R_L)I}{R_B + [(1-w)\hat{R} + wR_L - R_B]I} + \frac{(-\hat{R} + R_H)I}{R_G + [(1-w)\hat{R} + wR_H - R_G]I}\right] +$$
$$\frac{1-\beta}{2}\left[\frac{(-\overline{R}' + R_L)I}{R_B + [(1-w)\overline{R}' + wR_L - R_B]I} + \frac{(-\overline{R}' + R_H)I}{R_G + [(1-w)\overline{R}' + wR_H - R_G]I}\right]$$

When $w = 0$, the cash returns will converge across the two states.

$$\frac{\hat{R}}{R_B} = \frac{\hat{R}}{R_G} \quad \text{and} \quad R_B = R_G \equiv R_S$$

The first order derivative of w then reduces to the following.

$$\left[\frac{\beta}{2}\frac{R_L + R_H - 2\hat{R}}{R_S + (\hat{R} - R_S)I} + \frac{1-\beta}{2}\frac{R_L + R_H - 2\overline{R}'}{R_S + (\overline{R}' - R_S)I}\right]I \tag{A.16}$$

Similarly, one can calculate the first order condition of I,

$$\frac{\beta}{2}\left[\frac{(1-w)\hat{R} + wR_L - R_B}{R_B + [(1-w)\hat{R} + wR_L - R_B]I} + \frac{(1-w)\hat{R} + wR_H - R_G}{R_G + [(1-w)\hat{R} + wR_H - R_G]I}\right] +$$
$$\frac{1-\beta}{2}\left[\frac{(1-w)\overline{R}' + wR_L - R_B}{R_B + [(1-w)\overline{R}' + wR_L - R_B]I} + \frac{(1-w)\overline{R}' + wR_H - R_G}{R_G + [(1-w)\overline{R}' + wR_H - R_G]I}\right] = 0$$

When evaluated at $w = 0$, it reduces to

$$\frac{\beta}{2} \frac{2\hat{R} - 2R_S}{R_S + (\hat{R} - R_S)I} + \frac{1 - \beta}{2} \frac{2\overline{R}' - 2R_S}{R_S + (\overline{R}' - R_S)I} = 0. \tag{A.17}$$

Multiply (A.17) by I and add to expression (A.16), one can transform the first order derivative of w into the following.

$$\left[\frac{\beta}{2} \frac{R_L + R_H - 2R_S}{R_S + (\hat{R} - R_S)I} + \frac{1 - \beta}{2} \frac{R_L + R_H - 2R_S}{R_S + (\overline{R}' - R_S)I} \right] I \tag{A.18}$$

Note that the denominators are consumption at $t = 1$ and $t = 2$ and they are positive. Given the common numerator $(R_L + R_H - 2R_S)$, a sufficient and necessary condition for (A.18) to be positive is

$$\frac{R_L + R_H}{2} = \overline{R} > R_S. \tag{A.19}$$

In order to solve for the endogenous R_S at $w = 0$, note that the first order condition of I and the market clearing conditions constitute of a system of two equations and two unknowns, I and R_S. To solve it, note that first order condition of I implies

$$I = -\frac{\beta(\hat{R} - R_S) + (1 - \beta)(\overline{R}' - R_S)}{(\hat{R} - R_S)(\overline{R}' - R_S)} R_S$$

And the market clearing condition implies

$$I = \frac{(1 - \beta)R_S}{(1 - \beta)R_S + \beta\hat{R}}$$

Equalizing the two expressions, we will have the expression of R_S.

$$R_S = \frac{\beta R_H + R_L}{1 + \beta} \frac{(1 + \beta^2)R_L + (2\beta - \beta^2 + 1)R_H}{(2 - \beta + \beta^2)R_L + (3\beta - \beta^2)R_H}$$

The expression of $R_S - \overline{R}$ then follows.

$$R_S - \overline{R} = \frac{-2\beta R_H^2 - \left[(1 - \beta^2) + \beta(1 + \beta^2)\right]R_L^2 - \left[(1 + \beta)^2 + \beta(1 + 2\beta - \beta^2)\right]R_L R_H}{2\left[(1 + \beta^2)R_L + (3\beta - \beta^2)R_H + (1 - \beta)R_L\right](1 + \beta)}$$

Note that for $\beta \in [0, 1]$ the denominator is positive. Concerning the numerator, all the coefficients in front of returns are negative. Therefore, we prove $\overline{R} > R_S$ and $w^* > 0$. The investors will choose optimally a positive exposure to the aggregate risk.

Table 4: Variable definitions and data sources

Variable name	Definition	Data source
Bank Size	logarithm of total assets	Call report
Returns on Assets	returns on asset, (net income/total assets)*100	Call report
Risk Weighted Asset/Asset	(risk weighted assets/total assets)*100	Call report
Total Asset/Equity	total assets divided by equity	Call report
GDP Growth	the growth rate of real GDP	Bureau of Economic Analysis
Unemployment Rate	unemployment rate	Bureau of Labor Statistics
Repo Funding/Liabilities	(repo funding/total liability)*100	Call report
Repo Rate	U.S.overnight repo rate (general collateral)	ICAP
Deviation from Taylor's Rule	federal fund rate minus Taylor's rate	Author's calculation

Taylor rates are calculated as $i = r^* + \pi^* + 1.5(\pi - \pi^*) + 0.5y$, where π is cpi, y is output gap, π^* is the inflation target and r^* is the long-run level of the real interest rate. Following Taylor (1993), the response coefficients are set at 0.5 and 1.5. The inflation target is assumed to be 2%; and the long-run real interest rate 2%. CPI data are retrieved from Bureau of Economic Analysis and the quarterly data on output gap are retrieved from Oxford Economics.

Table 5: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Bank Size	15.251	1.439	13.665	21.357	6266
Returns on Assets	0.721	0.435	-2.201	2.477	6266
Risk Weighted Asset/Asset	75.642	11.403	29.957	99.878	6266
Total Asset/Equity	12.06	2.945	4.105	24.269	6266
GDP Growth	0.724	0.386	0.048	1.69	6266
Unemployment Rate	5.362	0.528	4.4	6.100	6266
Repo Funding/Liabilities	4.137	5.275	0	46.095	6266
Repo Rate	2.486	1.545	0.92	5.25	6266
Deviation from Taylor's Rule	-3.444	1.395	-5.298	0.496	6266

Appendix B   Data Appendix
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